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ARRESTING CIRCLES IN FORMAL DIALOGUES*

The study of the informal fallacies has an importance acknowledged by its place in both ancient and contemporary logic texts. Unfortunately, their study produces what Hamblin [3] has called the Standard Treatment, bereft of theory, laden with dogma and hoary illustrations appropriated from Aristotle, a corpus that often seems more to perpetrate the fallacies than to explicate them. Recently, however, a few writers have been seeking for some basic theoretical understanding of what these fallacies are, and we hope here to add something to this effort with some remarks about the *petitio principii*. Notwithstanding invitations occasioned by ambiguities which lurk in the title of this paper, our aim is not to report on dialogical circles we have known and loved, but rather to say something about how they may be stopped.

1. BACKGROUND

It is of some interest to determine the extent to which circular reasoning can be studied by means of three distinct kinds of system. They are: (i) Systems of doxastic–epistemic logic such as may be found in the work of Kripke [6], Hintikka [4], and Wu [13]; (ii) systems of formal dialectic or formal dialogues as represented by Hamblin [3] and Lorenzen [7]; and (iii) such non-intensional, non-game-theoretic standard approaches as are afforded by first-order logic. De Morgan [2], pp. 254ff., presents the first systematic case for an approach to the *petitio* in the style of (iii), and a note of Hoffman [5] pursues a similar method. *De Morgan's Thesis*, as we shall call it, is that every genuine instance of the *petitio* is reducible to the case in which a conclusion is identical to some premiss or conjunctive part of a premiss. The trouble with this, however, is that examples abound of clearly circular arguments in the form of the disjunctive syllogism, yet in which the conclusion is not identical to any premiss or conjunctive part thereof.

It is ventured in Sanford [8] and in Woods and Walton [11] that De Morgan's treatment is inadequate to the *petitio*, and in Woods and

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Walton [10] that a satisfactory understanding of this fallacy involves, essentially, epistemic–doxastic notions. One example of an epistemic circularity condition, let us call it (CDE), might provide that an argument is circular in relation to a given person if, and only if, in order to know that some premiss (or conjunctive part of a premiss) is true, the person to whom the argument is directed must know that the conclusion is true.

Still, at least one philosopher has felt (Barker [1]) that begging the question always presupposes a context of disputation, a setting in which there is a controversy over one or more issues, and that outside such a context, the *petitio* does not occur. Such a preference for a dialogical treatment of the fallacy probably arises from the suspicion that in Approach (i) there lie unacceptable presumptions of psychologism. After all, it might be protested (though without too much force we would suggest), the overtly 'subjective' parameters of Approach (i) are not likely to capture the essence of what is, let us remember, a logical matter.

However, it seems to us likely that the best account of begging the question will incorporate elements of all three approaches. In [10] and [11] we have pursued the *petitio* mainly from the perspective of (i), namely simple frameworks of doxastic logic, though there was also some informal attention paid to dialogical considerations. Here, however, our focus will be, in the manner of Approach (ii), more single-mindedly dialogical. Our task will be to determine whether the *petitio* can adequately be characterized in certain kinds of dialectical games: one constructed by Hamblin for this purpose; another, constructed by us following Kripke's semantics for intuitionistic logic; and still another, which is a simplification of the second.

2. THE GAME H

'Why-Because-System-with-Questions,' designed by Hamblin [3] 265 ff., affords a possible solution of problems of organization of commitment. There are two participants, White, and Black, each of whom has a commitment-store containing a finite set of statements, and each participant may add or delete commitments to or from his own store according to rules set out below. Let C be a's commitment-store, where a is a participant. Then, for one rather natural interpretation of the notion, one we have employed elsewhere, one might say that $x \in C$ iff $(\exists y)(Bay \wedge y \vdash x)$, with '$B$' the belief-operator. Let us say at once that this is *not* the interpretation that we wish to employ for Hamblin's game. For that purpose commitment-stores are finite, whereas on the current interpretation they are infinite whenever they are non-empty. The language of this game is basically first-order, restricted to a finite set of atomic statements. Axioms of the system are contained in the initial commitment-stores of both participants. White moves first, Black responds, and each continues in turn to make one move at a time. The capital letters S, T, U, . . . are variables of the metalanguage for statements.

Locutions may consist of the following types:

(i) ⌜Statement S⌝ or, in certain special cases, ⌜Statements S, T⌝.

(ii) ⌜No commitment $S, T, \ldots, X$⌝, for any finite number of statements $S, T, \ldots, X$ (one or more).

(iii) ⌜Question $S, T, \ldots, X$?⌝, for any number of statements (one or more).

(iv) ⌜Why S?⌝, for any statement S other than a substitution-instance of an axiom.

(v) ⌜Resolve S⌝.

Two categories of rules are given; locution rules and rules of commitment-store operation.

Locution Rules:

S1. Each speaker contributes one locution at a time, except that a 'No commitment' locution may accompany a 'Why' locution.

S2. ⌜Question $S, T, \ldots, X$?⌝ must be followed by
(a) ⌜Statement-$(S \vee T \ldots \vee X)$⌝ ('-' for negation)
or (b) ⌜No commitment $S \vee T \vee \ldots \vee X$⌝
or (*c*) ⌜Statement S⌝ or
⌜Statement T⌝ or
―――――― or
⌜Statement X⌝
or (d) ⌜No commitment $S, T, \ldots, X$⌝

S3. ⌜Why S?⌝ must be followed by
(a) ⌜Statement $-S$⌝
or (b) ⌜No commitment S⌝
or (c) ⌜Statement T⌝ where T is equivalent to S by primitive definition.
or (d) ⌜Statements $T, T \supset S$⌝ for any T.

S4. ⌜Statements S, T⌝ may not be used except as in 3(d).

S5. ⌜Resolve S⌝ must be followed by
(a) ⌜No commitment S⌝
or (b) ⌜No commitment $-S$⌝.

Commitment-store operation

C1. ⌜Statement S⌝ places S in the speaker's commitment store unless it is already there, and in the hearer's commitment-store unless his next locution states ⌜$-S$⌝ or indicates 'No commitment' to S (with or without other statements); or, if the hearer's next locution is ⌜Why S?⌝, placement of S in the hearer's store is suspended until the hearer explicitly or tacitly accepts the proferred reasons (see below).

C2. ⌜Statements S, T⌝ places both S and T in the speaker's and hearer's commitment-stores under the same conditions as in C1.

C3. ⌜No commitment $S, T, \ldots, X$⌝ deletes from the speaker's commitment-store any of $S, T, \ldots, X$ that are in it and are not axioms.

C4. ⌜Question $S, T, \ldots, X$?⌝ places the statement $S \vee T \vee \ldots \vee X$ in the speaker's store unless it is already there, and in the hearer's store unless he replies with ⌜Statement $-(S \vee T \vee \ldots \vee X)$⌝ or ⌜No commitment $S \vee T \vee \ldots \vee X$⌝.

C5. ⌜Why S?⌝ places S in the hearer's store unless it is there already, or he replies ⌜Statement $-S$⌝ or ⌜No commitment S⌝.

It is of some importance that in (H) commitment-stores are not closed under the classical logical operations. Hamblin considers requiring that the statements in a commitment-store be *consistent* (p. 263f.) but rejects this because "consistency presupposes the ability to detect even very remote consequences of what is stored, and this would itself make nonsense of certain kinds of possible dialectical application." On the other hand, Hamblin suggests (p. 264) that certain "very immediate" consequences of a commitment might also be regarded as commitments in a given system. The general idea therefore seems to be that (H) can be regarded as a base system upon which closure requirements of various strengths might be built up for various purposes of application.

There is another important general aspect that calls for comment in understanding the motivation of (H): the retraction of commitments by a participant is allowed. That is, statements may be deleted from, as well as added to commitment-stores at appropriate moves. Particularly difficult questions of degrees of closure of commitments under logical operations concern retraction. What is to happen if a participant retracts commitment to T or even replaces it by $-T$ when he is committed to both S and $\ulcorner S \supset T \urcorner$? Again, Hamblin would have it that specific rules need to be laid down in specific systems to deal with this sort of situation.

Hamblin in defending the base system (H) stresses that a commitment is not to be thought of as a 'belief' of the participant who has it, and he disavows any implication that the interest or point of commitment-stores is psychological. It is well to notice also that [3, 260 ff.] develops a formal version of the *Obligation Game* which has no provision for retraction of commitments. It is clear from this treatment that interesting formal games of dialogue *without retraction* can be constructed. We should also point out that while such games might have wide and various applications, Hamblin's primary purpose is to reveal structures of argument that might throw some light on forms of argument relevant to the study of the traditional informal fallacies.

3. CIRCLE GAMES

Hamblin discusses in [3, 268 ff.], various modifications and additions to both sets of rules for various purposes, but since our interest here is in the representation of circular reasoning, two rules are of special importance. The first is a rule for when the 'Why?' proposer is regarded as inviting his opponent to convince him:

(W) ⌜Why S?⌝ may not be used unless S is a commitment of the hearer and not of the speaker.

Otherwise 'Why?' is academic. The second rule is specifically designed to block circular reasoning:

(R1) The answer to ⌜Why S?⌝, if it is not ⌜Statement $\neg S$⌝ or ⌜No commitment S⌝, must be in terms of statements that are already commitments of both speaker and hearer.

Following van Dun [12, p. 110], dialogues will be represented by means of diagrams (see also Stegmüller [9] and Lorenzen [7]). The column on the left indicates the moves of White, those on the right the responses of Black. Pairs of moves are numbered on the left, and sequences of moves are set out using the method of nested sub-diagrams of tableaux.

The two most elementary forms of circular reasoning realizable in H can be represented by these dialogues.
A, B, and C are atomic statements,

	WHITE	BLACK
(1)	Why A?	Statements $A, A \supset A$.

	WHITE	BLACK
(1)	Why A?	Statements $B, B =_{df} A$
(2)	Why B?	Statements $A, A =_{df} B$.

Some other sequences, not mentioned by Hamblin, also represent kinds of circular argument.

	WHITE	BLACK
(1)	Why A?	Statements $B, B \supset A$
(2)	Why $B \supset A$	Statements $A, A \supset (B \supset A)$.

	WHITE	BLACK
(1)	Why A?	Statements $B, B \supset A$
(2)	Why B?	Statements $A, A \supset B$.

The latter sequence represents a paradigm of circular argument, and in the sequel we will call an argument having this form *a circle game*. In a circle game it is possible to have a third step, C, intervening between the beginning and the end of the circle, as follows.

	WHITE	BLACK
(1)	Why A?	Statements $B, B \supset A$
(2)	Why B?	Statements $C, C \supset B$
(3)	Why C?	Statements $A, A \supset C$.

Of course, the same form of sequence can be carried through to n steps, allowing for many intervening steps before the circle is closed by Black.

	WHITE	BLACK
(1)	Why A?	Statements $A_1, A_1 \supset A$
(2)	Why A_1?	Statements $A_2, A_2 \supset A_1$
.	.	.
.	.	.
.	.	.
(k)	Why A_{n-1}?	Statements $A_n, A_n \supset A_{n-1}$
(k + 1)	Why A_n?	Statements $A, A \supset A_n$.

The second and third sequences we looked at above may be likewise expanded. Though analogous things could be said about these other two kinds of games, in what follows we confine our remarks to circle games.

4. ADEQUACY OF H + (W) + (R1) FOR REPRESENTING THE PETITIO

How do (W) and (R1) block the *petitio*? Consider a two-statement circle game (the fourth game we looked at above). When Black responds, $\ulcorner B, B \supset A \urcorner$ at step (1), it is required by (R1) that both statements be in the commitment-store of both Black and White. Thus by (W), White's move at (2) is illicit, since he may ask $\ulcorner$Why B?$\urcorner$ only if B is not a commitment of his.

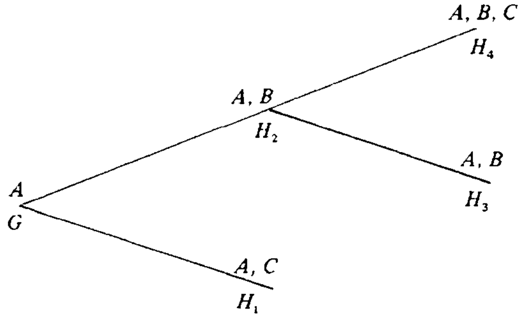
However, it would appear that an interesting form of sequence can be constructed without violence to H + (W) + (R1) but which has instances that may plausibly be interpreted as circular. Let us see. For the reader's convenience we set out the initial commitment-store of each participant in brackets at the head of the tableau. A superscript indicates at which step an addition is made; a stroke indicates deletions; and a superscript at the head of the stroke marks the step at which that statement was removed from the store.

	WHITE $[A \supset B, B \supset A, A^2, \not{B}^3]$	BLACK $[A, B, A \supset B, B \supset A, C]$
1.	Why A?	Statements $B, B \supset A$
2.	Statement A	Statement C
3.	No commitment B; why B?	Statements $A, A \supset B$
4.	Statement B	

Description: Black accepts the truth of A and B and also their equivalence (mutual material conditional). White accepts the mutual implication, and accepts B. White asks $\ulcorner$Why A?$\urcorner$ and Black responds by citing B and $\ulcorner B \supset A\urcorner$, each of which is accepted by both parties. White concedes A. Black then moves on to something else. But then White 'gets the jitters' and (see Rule (W)) retracts his commitment to B, asking $\ulcorner$Why B?$\urcorner$. Black responds by citing A and $\ulcorner A \supset B\urcorner$, both of which are now in the commitment-stores of both parties. White concedes B. Note that at step 3 White is inconsistent, i.e., his commitment-store contains A and $A \supset B$ while at 3 he retracts commitment to B. To be consistent, he should also retract commitment to A or $A \supset B$. However, he moves back to consistency at 4. This he may do: commitment-sets are not closed under logical operations.

One way of understanding the dialogical sequence of Section 4 is that White can know that A is true at 2 only on the basis of B, as set out in Black's response at 1. Then by (CDE) (see Section 1) White's knowing that B on the basis of A closes the circle. We now develop this idea using the intuitionistic semantics of Kripke [6].

The model Kripke uses is that of the *tree*, such as the example given just below,[1] where G is the origin, the unique bottom node, and H_1, H_2, H_3, H_4 are ascending nodes of the tree. A sentence letter, A, B, C, is verified at a point if written above that node; if omitted, it is unverified at that point. Thus in the example below, A is verified at G but B is unverified at G:



The nodes H_i represent points of time or 'evidential situations' at which times we may have various pieces of information. If we have enough information to ground a particular proposition A at a point H, we say A is verified at H; if we lack enough information to ground a proposition A at H, we say A is not verified at H.

The Kripke tree structure as a semantic representation of the idea of advancing states of knowledge may be illustrated with reference to the example above.[2] One possibility is that we may remain 'stuck' at G without gaining any new information. But it is possible that we will gain enough new information to 'jump' to point H_1 where we have a verification of C, as well as of A, or likewise to H_2, where we will have verified both A and B. Similarly, we could be 'stuck' at H_2 an arbitrarily long time, or advance to H_3 or H_4. Note that H_2 is significantly different from H_3 even though both A and B are verified at both points. As long as we remain at H_2, the possibility of advancing to H_4 remains open, but if we are at H_3 we have gained enough information to exclude every verifying C.

What sorts of connectives do we have in a Kripke model? Disjunction and conjunction are exactly analogous to the classical truth-functions. Negation and implication however are quite different.[3] To assert $\ulcorner \neg A \urcorner$ at a point H, we need to know at H not only that A has not been verified at H, but that A cannot possibly be verified at any later time, no matter how much new information is gained. Accordingly, $\ulcorner \neg A \urcorner$ is said to be verified at H if, and only if, A is not verified at every point H' accessible to H. To assert $\ulcorner A \supset B \urcorner$ at H, we need to know that at any later situation, where we have a verification of A, we also have a verification of B. Thus $\ulcorner A \supset B \urcorner$ is said to be verified at H if, and only if, for every point H' accessible to H, A is not verified at H or B is verified at H'.

Formally, the model theory is as follows. An (intuitionistic) *model structure* is an ordered triple $\langle G, K, R \rangle$, where K is a set, G is an element of K and R is a reflexive and transitive relation on R. An (intuitionistic) *model* on a model structure is a binary function $\emptyset\,(P, H)$, where P ranges over arbitrary proposition letters, and H ranges over elements of K, whose range is the set {T, F}, and which satisfies the condition: if $\emptyset\,(P, H) = \mathrm{T}$ and HRH' $(H, H' \in K)$, then $\emptyset\,(P, H') = \mathrm{T}$. We get from atomic propositions to formulae $A, B, C, \ldots$ of the propositional calculus by the following clauses.

(a) $\emptyset\,(A \wedge B, H) = \mathrm{T}$ iff $\emptyset\,(A, H) = \emptyset\,(B, H) = \mathrm{T}$; otherwise $\emptyset\,(A \wedge B, H) = \mathrm{F}$.

(b) $\emptyset\,(A \vee B, H) = \mathrm{T}$ iff $\emptyset\,(A, H) = \mathrm{T}$ or $\emptyset\,(B, H) = \mathrm{T}$; otherwise $\emptyset\,(A \vee B) = \mathrm{F}$.

(c) $\emptyset\,(A \supset B, H) = \mathrm{T}$ iff for all $H' \in K$ such that HRH', $\emptyset\,(A, H') = \mathrm{F}$ or $\emptyset\,(B, H') = \mathrm{T}$; otherwise $\emptyset\,(A \supset B, H') = \mathrm{F}$.

(d) $\emptyset\,(\neg A, H) = \mathrm{T}$ iff for all $H' \in K$ such that HRH', $\emptyset\,(A, H') = \mathrm{F}$; otherwise $\emptyset\,(\neg A, H) = \mathrm{F}$.

A formula A of propositional calculus is called *valid* iff $\emptyset\,(A, G) = \mathrm{T}$ for every model on a model structure $\langle G, K, R \rangle$.

The notion that there is an epistemic priority in proving propositions may be represented in a Kripke model, and consequently, one kind of circularity can be expressed in the Kripke framework. Given an argument of the form '$\ulcorner A, A \supset B \urcorner$ is verified at H', where A represents the premiss-set and B the conclusion, we might say that the argument does not beg the question if, and only if $\ulcorner B \supset A \urcorner$ is not established at H. We oversimplify in that it is usually one premiss or part of a premiss, and not the whole premiss-set, that begs the question, but this complication presents no real difficulty. Thus our condition provides, in effect, that the conclusion of a non-circular argument must be established at some point in the evidentiary process after premissory verification. The condition may not be adequate to the fullest understanding of *petitio* in all respects – and we shall not attempt to show generally that it is (see [10]) – but it does give us a sufficiently compelling condition to make our demonstration of the incompleteness of Hamblin-games in an appropriate Kripke model of some interest to the concerns of this paper.

Consider now the following representation in a Kripke model of the dialogue of Section 4. By Black's move at 1, B is true at G, and B and A are true at H.

$$\begin{array}{ccc} B & \_\_\_\_\_\_ & B, A \\ G & & H \end{array}$$

White's move at 2 is presumably to accept the truth of A at H. Up to this point argument $\ulcorner A, A \supset B \urcorner$ is non-circular, since A is established at H, a point later than G. But Black clearly violates the circularity condition at 3, since in this case,

$$\begin{array}{ccc} B, A & \_\_\_\_\_\_ & B, A \\ G & & H \end{array}$$

whether the conclusion is A or B, in neither case is the conclusion established at a point earlier than the premiss. Thus in the Kripke model Black commits the fallacy of *petitio* at 3. Of course we assume throughout that White and Black intend to assert their various statements $A, B, \ulcorner A \supset B \urcorner$ and $\ulcorner B \supset A \urcorner$ at the same point, G. Otherwise, the fallacy is more of equivocation than *petitio*. It begins to appear, then, that the game H + (W) = (R1) does indeed permit a fallacy of *petitio* on our version of the Kripke interpretation. We might also construe this result as additional confirmation of a conjecture of [10] and [11] namely, that *petitio* has the epistemic aspect of Approach (i), although, as we shall have occasion to say below, this remains very much a conjecture.

5. CUMULATIVENESS

Whether Hamblin's rules are in fact adequate to forestall all circular reasoning requires a closer examination of similarities and differences between the system (H) and Kripke's intuitionistic semantics [('K') let us say for short].

One outstanding difference between (H) and (K) is that the latter is essentially cumulative or incremental in a way that the former is not. On a Kripke tree, if a proposition is verified at a point (node) then that proposition must remain verified at every succeeding point (at every related node). Whereas in the game (H), if a statement is in the commitment-store of a participant at a given point (or move of that participant) it does not follow that the statement must remain in his commitment-store at every succeeding move he might make (at every next line of the tableau). (H) is not essentially cumulative because, as we have seen, (H) is the sort of game that, unlike the Obligation game or other games of dialogue, allows for retraction of commitments. Thus it would seem that any structure that consists of an ordered set of points can be thought of as having the property of essential cumulativeness or not, regardless of whether the points are interpreted as possible worlds or moves in a game, regardless of whether the framework is epistemic or game-theoretic. To define an appropriately general notion of cumulativeness, then, we need the following ingredients: (1) a set of points $w_i \in W$, (2) an ordering relation $<$ on the w_i, (3) a language L, a set of propositions or statements $A, B, C, \ldots$, and (4) a function f that takes a pair $\{w_i, A\}$ onto a set $\{1, 0\}$. The idea behind (4) is that a given proposition can either obtain or not (have the value 1 or 0) at any given point w_i. Now the definition can be given: a system $\langle W, <, L, f\rangle$ is *cumulative* if, and only if, for any two points $w_i, w_j \in W$, for any proposition A, if A has a given value (1 or 0) at w_i then A has the same value of w_j if $w_i < w_j$.

It is now easily seen that the Kripke system is cumulative whereas the Hamblin system (H) is not. The proofs in both cases are straightforward once the basic idea is sketched out, so we simply offer the sketches. First we show that the Kripke system is cumulative. A relation $<$ defines what Kripke calls a *tree* over a set W if for any $w_i, w_j, w_k \in W$, if $w_i < w_k$ and $w_j < w_k$ then $w_i = w_j$. Then we let f correspond to Kripke's $\emptyset$ and L to Kripke's intuitionistic calculus defined by the conditions (a)–(d) for $\wedge$, $\vee$, $\supset$, and $\neg$ given in [6, p. 94]. Then we note that Kripke sets down for arbitrary proposition letters P and for his binary function $\emptyset(P, H)$, where H is the set of 'nodes' of the tree and $\emptyset$ corresponds to our f, the following condition: if $\emptyset(P, H) = \mathrm{T}$ and HRH' $(H, H' \in K)$, then $\emptyset(P, H) = \mathrm{T}$. Then he shows that once this property has been stipulated for a propositional letter (atomic formula) it follows for the complex formulae formed by clauses (a)–(d). So obviously Kripke's system is cumulative in our sense, for once a formula A is verified at a node H_k it will be verified (i.e., have the value T, corresponding to our value 1) at every node 'beyond' H_k in a given tree structure.

We show (H) to be non-cumulative by letting each w_i correspond to a line in the tableau of a dialogue. Then we can think of the lines ordered by the relation $<$ on W, i.e., 'w_j is a later line than w_i'. Then we take a fixed participant, a, and we let $f(A, w_i) = 1$ be satisfied in relation to a if the statement A is a commitment of a at line w_i of the dialogue. And we let $f(A, w_i) = 0$ be satisfied in relation to A if the statement A is not a commitment of a at line w_i of the dialogue. Obviously then there is no problem constructing a model of a dialogue permitted by the rules of (H) where A is a commitment of some participant at a given step, and then A is no longer a commitment of the same participant at some later step. Indeed, our problematic circle game of Section 4 provides one such example.

Now that we are equipped with a reasonably clear notion of cumulativeness, let us look back to the troublesome dialogue of Section 4. We can now pose the following formidable objection to the thesis that there is a circle in this dialogue according to the criterion of circularity constructed in the Kripke model in Section 5. The objection is this: there seems to be a circular sequence in the dialogue only if one sees it through the rose-coloured glasses of the essentially cumulative scaffolding of circularity typified by the framework of Section 5. The suggestion is that once we see that White retracts his commitment to B at 3, and that this is a legitimate move, the intuition that there is a circle in the dialogue quickly pales into unconfidence. That is, we may strictly speaking have a circle, but it would be nonsense to call it a fallacy. After all, in a non-incremental dialogical game, White has the right to retract his commitment to B at 3, and then change his mind *again* at 4. White could be accused of vacillating, but this does not amount to *petitio*.

This line of thought throws new light on the problem because the operative distinction changes from epistemic/dialogical to cumulative/non-cumulative. The critical factor in whether or not a *petitio* is committed is not whether the system is designed to model dialogical exchanges or epistemic states but whether it is cumulative in regard to certain values that the propositions (statements) are said to have.

6. GROUNDEDNESS

Another reason for thinking the dialogue of Section 4 is somehow circular is that one might naturally read White's statement that A at 2 is a *response* to the move of Black at 1 in the sense that White accepts A *on the basis that* B. Then of course later at 4, when White appears to accept B *on the basis that* A, the circle is closed. The fact is, however, that nothing in the rules of (H) tells us that a move like White's statement at 2 constitutes a response *on the basis* of a statement set out in a previous line. Hamblin, let us remember, disavows such psychological assumptions. Nevertheless, it is open to us to add a notion of groundedness to the dialogical steps of (H) in an effort to capture this intuition. Could we not define groundedness, for example, as follows: a statement A *is grounded on* a statement B in a Hamblin game if, and only if, A is a move of some participant at step k of a dialogue and $\ulcorner B, B \supset A \urcorner$ is a move of another participant at step $k - 1$? Then a dialogue could be said to be circular if two statements are grounded on each other. By this criterion, our specimen in Section 4 is indeed circular. The relation of groundedness would presumably be transitive, and the relation of non-circular groundedness would be irreflexive and anti-symmetrical. Groundedness could also be defined on Kripke models as follows: Q is *grounded on* P at H in a Kripke model if, and only if, P is verified at some node H and Q is verified at any node H'' such that (i) HRH'', and (ii) there is no node H' such that HRH' and $H'RH''$ for distinct nodes H, H', H''. In other words Q is grounded on P if Q is verified at the next node to one P is verified at. By the use of definitions of this sort, we seem to be able to reinforce the argument that at least one form of circularity in Section 4 can be identified.

Our own view is that while we are prepared to concede that something might be done with the notion of groundedness in the Kripke type of cumulative framework, we are hard-pressed to see what such a notion could amount to in a game that allows for retractions. The problem is, how do we deal with step 3 (taking our sequence of Section 4 again as example)? When White retracts B, are we still to say that his acceptance of A is 'grounded on' B? If his acceptance of A is grounded on something he now rejects, shouldn't he also reject A? Well, perhaps, if he 'remembers' and if he hasn't discovered other grounds for A in the meantime, or if he had other reasons for accepting A all along that now disincline him to reject A. Each of these possibilities yields a new way of looking at groundedness, and each in turn canfbe expected to modify our intuitions as to whether the dialogue is circular. What we seem to have here is a host of new parameters concerning factors such as 'total evidence' versus 'partial evidence', factors that could be very elusive to define. In short, defining groundedness of A on B relative simply to the statement of $\ulcorner B, B \supset A \urcorner$ by the other participant at the previous step of a dialogue seems to us too thin a definition properly to support any single, clear intuition that the dialogue is circular or not.

7. A LINEAR SIMPLIFICATION

It is perhaps worth noting that in systems such as (K) one can have both strong and weak negation. This suggests that there are two distinct classes of possible connectives there definable: (a) the classical connectives, namely, $\vee$ and $\wedge$ as defined by Kripke *plus* weak negation, i.e., $\neg p$ at H has the value T if, and only if, p does not have the value T at H) and the classical conditional, $p \supset q$ at $H =_{df} \neg(p \wedge \neg q)$ at H; (b) the intuitionistic connectives, namely the conditional $\rightarrow$ and the negation $\sim$ defined by Kripke: (1) $p \rightarrow q$ is true at H if, and only if, q has the value T at every point accessible to p at H, (2) $\sim p$ is true at H if, and only if, p has the value T at no point accessible to H. Notice that all the classical connectives are definable exclusively in reference to a single point H, whereas the intuitionistic connectives are defined in reference to a *spread* of points H_i. This is rather neat; for as long as we are at one single point (evidential situation) and are abstracting from the cumulative epistemic aspect, we have classical logic. But as soon as we bring in the epistemic aspect, involving the transition from a given point to a set of related points in the evidential progression, we get into a richer logic (which calls to mind, by the way the analogy between IC and S4). In this richer logic we are essentially concerned with sets of points $\geqslant 2$. Thus it doesn't so much matter what the individual connectives are as whether they can be defined exclusively in reference to a *single* point in the model. It is this latter property that seems to characterize their essential intuitionistness (or epistemicness, if you like). This brings out the importance of the notion of cumulativeness again, because 'cumulativeness' requires for its expression at least a pair of points. And it underlines why, in classical alethic logic, we do not get a very good model of arguing in a circle, or of the direction of evidentiary inference.

We can now easily enough see that the *petitio* can be characterized in an even simpler framework than that given by Kripke. For consider the model of a set of points ordered on a line:

This structure is a Kripke tree with following condition: if H_iRH_j and H_iRH_k then $H_j = H_k$. In other words, it is a 'tree' but it is all trunk; no 'branching' is allowed (this is what mathematicians call a 'chain'). So suppose we have a set of points H_i ordered by a Kripke R-relation on a line or chain as above. Consider the following sort of business:

$$\begin{array}{cccc} p & p, q & p, q, r & \ldots \\ \hline H_0 & H_1 & H_2 & \ldots \end{array}$$

Now we adopt the rule, in picking out arguments over such an array, that only the 'new' letter is picked as conclusion at each stage. At stage H_1, by this rule, p is the premiss-set and q the conclusion. Plainly there is no circularity here. Next, at H_2, $\{p, q\}$ is the premiss-set and r the conclusion. Again, no circularity here. And so forth. The structure is essentially cumulative in a way that systematically prevents circularity. At each step the 'conclusion' is new, relative to the previous 'evidentiary situation'.

Notice too that Kripke's conditions rule out 'backward reasoning', e.g.,

$$\begin{array}{cc} p, q & p \\ \hline H_1 & H_2 \end{array}$$

but not 'circular reasoning' (no real evidentiary progression), is ruled out.

$$\begin{array}{cc} p & p \\ \hline H_1 & H_2 \end{array}$$

However it is easy to ban circles by simply adopting the rule above. Such a rule corresponds with our suggestion above for banning circles in the Kripke model. But it is interesting to note that cumulativeness (and consequently circularity) can be modelled linearly.

8. CONCLUDING REMARKS

It is not so much whether the system is epistemic or dialogical that tends to account for how well the fallacy of *petitio* can be modelled; the critical parameters would seem to pertain to such notions as cumulativeness and groundedness that might be defined in *either* type of system. Of course we have offered in these pages nothing like an exhaustive account of these notions, and it *might* turn out that the various categories have some well-defined correlations. For example, it might turn out that epistemic logic is best thought of as essentially cumulative and dialogical logic not. In the meantime, however, until more is known about the study and classification of these not very well defined families of systems, we shall have to declare the question open. What our work does suggest, however, is that instead of trying the two traditionally disparate methods of studying *petitio*, the epistemic and dialectical, more might be accomplished by studying their similarities and differences in a comparative way. We also think the model dialogue of Section 4 is an especially important specimen against which to test our theories. If nothing else, it is rich in intuitive conflicts.

It seems to us that, in modelling the *petitio*, cumulative systems are simpler. Consequently cumulative systems such as (K) yield a clear and relatively simple model of circular argument. When one extends this model to non-cumulative systems many complexities ensue, some of which may be expected to (and do) obscure the idea of *petitio*. Those who cleave to the relatively clear notion of circularity of Section 5 and who cannot bring themselves to understand the *petitio* in non-cumulative contexts may well persist in arguing that in the non-cumulative system (H) circles may occur. For it does seem clear that there is no effective way of prohibiting dialogues that, from a cumulative perspective on the *petitio*, are circular. Even so, the available evidence suggests that there are special difficulties in defining a workable concept of *petitio* adequate to non-cumulative systems. Whether such evidence should be regarded as chastening we do not venture to guess.

Still, there is some reassurance in recognizing that in the Kripke framework a useful theory of *petitio* can be formulated. We would suggest in as much as epistemic logic has an affinity for cumulative systems, we can have a theory of *petitio* that is epistemic in character, even if more exotic possibilities remain as potential competitors.

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NOTES

* We are indebted to participants of the Logic Seminar, at Victoria University of Wellington, New Zealand, for comments on an earlier draft read to that group by one author.

1 See [6], p. 97 ff.

2 See [6], p. 99.

3 See [6], p. 99 and p. 94.

REFERENCES

[1] John A. Barker, 'The Fallacy of Begging the Question', *Dialogue* XV (1976) 241–55.

[2] Augustus De Morgan, *Formal Logic*, London, Taylor and Walton, 1847.

[3] C. L. Hamblin, *Fallacies*, London, Methuen, 1970.

[4] Jaakko Hintikka, *Knowledge and Belief*, Ithaca, Cornell University Press, 1962.

[5] Robert Hoffman, 'On Begging the Question at Any Time', *Analysis* 32 (1972) 197–99.

[6] Saul Kripke, 'Semantical Analysis of Intuitionistic Logic I', *Formal Systems and Recursive Functions*, ed. by J. N. Crossley and M. A. E. Dummett, Amsterdam, North-Holland, 1965, pp. 92–130.

[7] Paul Lorenzen, 'Ein Dialogisches Konstruktivitatskriterium', in *Infinitistic Methods*, New York, Pergamon Press, 1961.

[8] David Sanford, 'Begging the Question', *Analysis* 32 (1972) 197–99.

[9] W. Stegmüller, 'Remarks on the Completeness of Logical Systems Relative to the Validity Concepts of P. Lorenzen and K. Lorenz', *Notre Dame Journal of Formal Logic* 5 (1964) 81–112.

[10] John Woods and Douglas Walton, '*Petitio Principii*', *Synthese* 31 (1975) 107–28

[11] John Woods and Douglas Walton, '*Petitio* and Relevant Many-Premissed Arguments', forthcoming in *Logique et Analyse*.

[12] Frank Van Dun, 'On the Modes of Opposition in the Formal Dialogues of P. Lorenzen', *Logique et Analyse* 57–58 (1972) 103–136.

[13] Kathleen Johnson Wu, 'A New Approach to Formalization of a Logic of Knowledge and Belief', *Logique et Analyse* 63–64 (1973) 513–525.