

*Informal Logic: The First International Symposium*, ed. J. Anthony Blair and Ralph H. Johnson, Inverness, California, Edgepress, 1980, 41-54.

***PETITIO PRINCIPII***  
**AND ARGUMENT**  
**ANALYSIS**

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The most acute problem in teaching and studying the field of informal fallacies is that lack of clear and theoretically adequate models of the fallacies makes it impossible to know or prove that what strongly seems to be a fallacy really is an argument that is incorrect, or in some sense invalid. Notoriously, it is also easy to get into unresolvable disputes about whether some evidently bad argument is an instance of one fallacy as opposed to another. But the deficiencies of what Hamblin [5] calls the Standard Treatment of the fallacies are well known. What is needed is some theory. At the same time, the unique value and appeal of the study of the fallacy domain is its potential applicability to the critical evaluation of argumentation, and therefore it is important that this theory should be strongly tied to the analysis of significant arguments.

A discouraging problem is that the quest for applicable theory might tend to take us far beyond the tidy domain of first order logic. Yet the history of the disarray that is the Standard Treatment suggests that there is little value in studying the fallacies until we achieve some general understanding of the underlying concepts of argument that are involved in the major informal fallacies. In this paper we will work towards trying to see how what is called *petitio principii* might be understood as a deficiency in arguments.

**I. THE STANDARD TREATMENT**  
**OF *PETITIO***

The history of the Standard Treatment of *petitio* exhibits a pair of dualisms. First, rooted in Aristotle's own treatment, there is the tendency to see it either as an epistemological phenomenon or as a dialectical (game-

theoretical) fallacy. In *Pr.An.* 64b 30 Aristotle treats *petitio* in light of his famous dictum that demonstration proceeds from what is more certain or better known: if a man tries to prove what is not self-evident by means of itself, he begs the question (*Pr.An.* 64b 37). The account of the fallacy here is epistemic. To be the question is to violate the epistemic principle of the priority in knowledge of the premisses over the conclusion in a demonstration. In the *Topics* however, the account is set in terms of contentious disputation between two or more parties. Begging the question is said to occur where a questioner, the party who is supposed to be arguing for a certain thesis, T, asks to be granted T as a premiss to be conceded by his opponent. This second account helps to explain the apparent peculiarity to modern ears of the phrases "begging the question" and "*petitio principii*." The second dualism is also a common historical theme and like the first, has survived through the ages into current logic textbooks. According to the *equivalence conception*,<sup>1</sup> an argument is said to be circular if the conclusion is assumed as a premiss, either as an exact equivalent or in a form so close to make the two statements virtually equivalent. As Copi puts it, "... two formulations can be sufficiently different to obscure the fact that one and the same proposition occurs both as premiss and conclusion" (*Introduction to Logic*, 4th ed., New York, Macmillan, 1972, p. 83). The problem with explicating this conception is that orthographic identity is too narrow a criterion, and logical equivalence does not seem to fit either.<sup>2</sup> The required notion of equivalence is elusive-perhaps it could be epistemological in nature. According to the *dependency conception*, an argument is said to be circular where the conclusion is required in order to establish some premiss. That is, according to this conception of a non-circular argument, one should be able to know that each premiss is true without having to infer it from the conclusion of the argument. The problem here is to explicate the required relation of dependency. As we have stated it here, obviously the notion of dependency appears to have an epistemic flavour.<sup>3</sup>

The above sketch may seem to indicate that the best places to look for some adequate theory to explain this phenomenon of *petitio* are either the dialectical structures proposed by Hamblin [5], Lorenzen [12], Mackenzie [13], and Rescher [15], or epistemic logic as developed by Hintikka [8] and developed in numerous subsequent studies. However, a general tendency when confronted by something new is to attempt to reduce it to first-order logic, and the historical basis for this possibility in regard to *petitio* is found in De Morgan's *Formal Logic* [1].

De Morgan [1, 254] preferred the purely alethic criterion that "strictly speaking, there is no formal *petitio principii* except when the very proposition to be proved, and not a mere synonyme of it, is assumed." This perhaps is what one might expect a logician to say, but the problem is that it scarcely seems applicable to a realistic doctrine of *petitio*. De Morgan however shrewdly backed up this posture with what John Woods and I in [20] call *De Morgan's Thesis*: no syllogism begs the question. We note that a syllogism has the following two properties: (1) it always has two premisses, and (2)

each premiss is logically independent of the other. Thus Woods and I suggested in [20] that De Morgan's thesis can be cast into an interesting modern form: no deductively valid argument with more than one premiss and no superfluous premiss begs the question. A premiss is *superfluous* in a valid argument if, and only if, the resulting argument is invalid if the premiss is deleted.

A main problem with De Morgan's thesis is that it appears open to some plausible counter-examples. For example, suppose I were to propose an argument to you in the form of a disjunctive syllogism [ $P \vee Q$ ,  $\neg P$ , therefore  $Q$ ] but clearly indicate that I mean to establish  $P \vee Q$  by appeal to  $Q$  through implication. Then I would have outrageously begged the question, but my argument is many-premissed and neither premiss is superfluous.

De Morgan might have replied that  $Q$  is really a "premiss" of the argument,<sup>4</sup> but was it or not? This raises an interesting question about how we identify premisses in the evaluation of argumentation. In fact it raises a lot of interesting questions about the applicability of logic to the evaluation of arguments, for, it would seem that we might really have two "arguments" in the present instance, each "linked" to the other.

$$Q \rightarrow P \vee Q, \neg P \rightarrow Q$$

The issue that is also raised therefore is: how in practice do we combine arguments in chain-like sequences to form *themata*, as Geach [4] calls them, from argument *schemata*? Whether De Morgan's thesis stands or falls seems to depend on these two unresolved issues in argument analysis.

This section would not be complete without remarking on J. S. Mill's famous *dictum* that all deductive logic commits *petitio*. Mill, in his *System of Logic* [14, 12Qf.] argued that the first premiss of the syllogism, "All men are mortal; Plato is a man; therefore Plato is mortal" presupposes the conclusion in the sense that we cannot be sure it is true unless we are already certain that the conclusion is true. If it is doubtful that Plato is mortal, Mill argued, it is at least as doubtful whether all men are mortal. De Morgan [1, 257] threw some doubt on this argument by pointing out that it overlooked the minor premiss. De Morgan, as we saw, in effect argued in flat contradiction to Mill that no valid syllogism begs the question. I suppose most of us would be inclined to avoid either of these extreme views by ruling that whether or not the syllogism above is circular depends on whether the context of argument includes (perhaps inductive) evidence for the major premiss that is independent of the conclusion, e.g. biological evidence of the mortality of animals. But this raises the perplexing question of the role of background evidence in the context of argument and consequently takes us back to the problem of what may or may not be considered a "premiss" of an argument.

## II. SOME RECENT WORK

Outside of first order logic, the two most likely candidates to throw some

light on the theory of circular argument would be epistemic logic, as studied by Hintikka [8] or Kripke [11], and the formal dialogues (dialectical games) of Hamblin [5]. The Hamblin game H is called a "Why-Because-Game-with-Questions" because it allows participants to advance questions of the form [Why A?] and it allows a respondent to reply with a locution of the form [Statements G,  $B \supset A$ , for any B.] This game has commitment-stores for all participants, and Hamblin [5, 268ff.] imposes a dual restriction on the operation of commitment-stores that, he claims, blocks *petitio* moves: ( $\alpha$ ) [Why A?] may not be used unless A is a commitment of the hearer and not the speaker; ( $\beta$ ) A "because" answer to [Why A?] must be in terms of statements B that are commitments of both participants (note: we henceforth restrict the number of participants to two, for ease of exposition). How does this block *petitio*? Consider a simple circle game.

WHITE	BLACK
(1) Why A?	Statements B, $B \supset A$
(2) Why B?	Statements A, $A \supset B$ .

White's move at (2) is always blocked by ( $\alpha$ ) because by ( $\beta$ ), B becomes a commitment of White by Black's response at (1). If Hamblin is right, it might seem that *petitio* can be handled, and perhaps also understood to some extent in the contexts of a game like H.

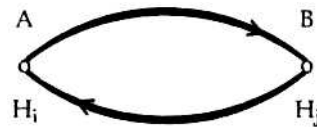
John Woods and I in [21] however have expressed some doubts about whether ( $\alpha$ ) and ( $\beta$ ) do effectively block circle games. The game H allows for retraction of commitments by participants, and problems may occur because a participant need not be committed to all logical consequences of his commitments. Consequently, as we have shown in [21], it is possible to construct dialogue-sequences in H that may be circular, or at least where intuitions clash or are uncertain on the question of whether there is a *petitio*.<sup>5</sup> These problems however are not my main worry, about the adequacy of rules like ( $\alpha$ ) and ( $\beta$ ) in games like H as means of capturing the notion of begging the question. The main worry is that ( $\beta$ ) is too strong in ruling out all justificatory moves except those utilizing premisses that are commitments of one's opponent. ( $\beta$ ) does not specifically represent the notion of begging the question at all.<sup>6</sup> To beg the question is to attempt to justify A either on the basis of A or on the basis of B, where B is also the basis of A as in the circle game above. Thus even if ( $\alpha$ ) and ( $\beta$ ) do block *petitio* in H, it is highly dubious whether they jointly characterize *petitio* as a concept of argument. Mackenzie [13] attempts to alleviate some of the problems of commitment-retraction in H in order to deal with *petitio*, but the basic problem remains that no set of rules of a game like H can adequately represent the essence of *petitio* exclusively by means of allowing or proscribing commitments of participants. *Petitio* pertains to the notion that Woods and I in [21] call groundedness, the idea that a statement A is *based on* some other statement B in the sense of being a justificatory response to [Why A?]. This notion is the heart of the dependency *petitio*.

In [21] a theory of *petitio* is offered in the framework of the Kripke semantics for intuitionistic logic [11]. In this semantics, we have a set of "evidential situations"  $H_i$  (possible states of knowledge at a given time) which are ordered as a tree. Statements  $A, B, C, \dots$  take on two values, "verified" or "not-verified" at these points  $H_j$ . Essentially, [21] rules that we have a *petitio* at a pair of points  $H_i, H_j$  where  $H_j$  is a "later point" in the ordering than  $H_i$  if there is no "new information" at  $H_j$  that had not already been verified at  $H_i$ , e.g.



Here we would have a *petitio* in the account of [21] because the "jump" from  $H_i$  to  $H_j$  represents no "new information" at  $H_j$ . We show in [21] that according to this conception of circular inference, circles can be constructed in  $H$  even with  $(\alpha)$  and  $(\beta)$ .

My present thinking is that while this theory of [21] does represent a worthwhile representation of the equivalence *petitio*, it does not adequately characterize the dependency *petitio*. In order to do that we would have to express this sort of structure in the model.



But this structure cannot be expressed in the Kripke modelling for the ordering of  $H_i$  is in the form of a tree, and it is well known that a tree is an acyclic graph. All this suggests that the Kripke model could be viewed as a special kind of graph and that *petitio* could be more generally studied in graph theory. Other factors will also suggest this possibility, and we will return to it in Section IV.

What of Hintikka's epistemic logic initially developed in [8]? The problem here is the well-known rationality assumption that a knower knows all the logical consequences of what he knows. Hamblin's  $H$  was not closed for consequences of commitments in any way and therefore had problems with commitment-retraction.<sup>7</sup> Hintikka's system is closed under *all* deductive consequences. One system is too weak, and the other too strong to be of much use in a realistic approach to argumentation. Indeed, it is specifically because of the strength of this rationality assumption that Hamblin [5, 238f.] rejects epistemic logic as an approach to the concept of argument appropriate to the study of the fallacies. It is not hard to see how Hamblin's remark is applicable in the case of *petitio*. As Aristotle pointed out, circular reasoning is fallacious in an epistemic context for the very reason that there is an order

in the epistemic weighting of propositions. But there can be no such ordering with regard to the pair {premises, conclusion} of a deductive argument if knowledge is closed under deductive consequence, for under that rationality assumption, both statements must be known equally by all knowers. In short, under Hintikka's assumption, there could never be anything fallacious about *petitio*.

Of course, in recent studies, e.g. [9], Hintikka has worked towards potentially more promising versions of epistemic logic by introducing the notion of a *surface tautology*. The idea is that a knower knows only the "obvious" consequences of what he knows, where obviousness is defined in terms of measures of the complexity of formulae. And then too, Hamblin [6] has tried to sort out some of the problems of assigning commitments in dialectical games by introducing the notion of an *immediate consequence* of a statement.<sup>8</sup> These are promising developments in needed theory.

### III.

#### FORMAL LOGIC AND THE LOGIC OF ARGUMENT

In discussing informal fallacies it is all too easy to acquiesce in the questionable assumption that the term Formal Logic (as capitalized by Hamblin) can be bandied about as though it referred to some nicely circumscribed referent. A glance at disputes in the foundations of mathematics will reveal the quick decomposition of any such easy assumption. In the teaching of logic in philosophy departments, what often passes for Formal logic is in fact first-order logic. A glance at heavily used textbooks like Copi's two well known primers however will indicate another fact, namely that first-order logic is often, perhaps usually, taught in philosophy departments as very much an applied logic, applied to the practice of evaluating arguments in natural language. Thus in practice, the domain of what passes for Formal Logic in many a curriculum contains much that is really of an informal nature. Let us try to be more specific. There are four basic tasks of the analysis of argumentation that the application of classical first-order logic presupposes as preliminaries.

First, classical first-order logic does not tell us whether the chunk of alleged argumentation that we are confronted with as *datum* really is an argument, as opposed to a reminder, threat, clarification, or whatever. That is, classical logic does not effect the tripartite classification *valid arguments/invalid arguments/not arguments at all*. It merely enables us to effect the former distinction. But in order to apply logic to the adjudication of an argument, we need to first decide whether we do really have an argument. This is a practical point, not merely a philosophical cavil, because a systematic sophist can always elude the force of logic by claiming, even if ingenuously, that what he advanced is not invalid because it is not an argument at all, but merely a clarification or something of the sort. Hamblin [5, 224f.] in his chapter on the concept of argument illustrates how such evasions can work in practice. So here is the first preliminary task for the application of classical

logic, the identification of arguments.

Second, given a pair of statements, classical logic contains no way of determining by any exact procedure which one is the "premisses" and which the "conclusion" of an argument. Deductive implication is not symmetrical, and therefore it can make a difference if we get this backwards. If we look to the texts, they tell us that the conclusion is the statement that is "established on the basis of" the premisses, which seems to be nothing more than a blatant appeal to our intuitions in the matter. Some texts appeal to so-called *indicator* words, like "therefore." However, "therefore" is not always present, and we have no decision procedure in classical logic for telling us when an indicator word is equivalent to a "therefore".

In regard to both tasks one and two, texts often correctly point out that the premisses and conclusion must be *designated* as such. This would seem to indicate that the methodology for such designations is external to the usual methodology given for first-order logic. This move may better circumscribe the two tasks, but it does not dispose of the need for them if logic is to be applicable to argumentation.

The third task is determining what *type* of argument we are confronted with. That is, assuming that there are, in addition to deductive arguments, things like inductive arguments, or perhaps even plausible arguments<sup>9</sup> that are neither deductive nor inductive, how are we to identify which type a given argument belongs to? The whole issue is rife with theoretical pitfalls and unclarities, but it is more than a purely philosophical puzzler. It is a practical question of the analysis of arguments. A systematic sophist could always escape a charge of deductive invalidity if pressed, by simply claiming that his argument was merely inductive, and escape the charge of inductive invalidity in turn by claiming mere plausibility. Conversely, a sophistical deductivist could always accuse his inductively arguing opponents of deductive invalidity. If there is no objective way of really determining which type an argument is, then there is no way of effectively barring such sophistical manoeuvres and counterattacks.

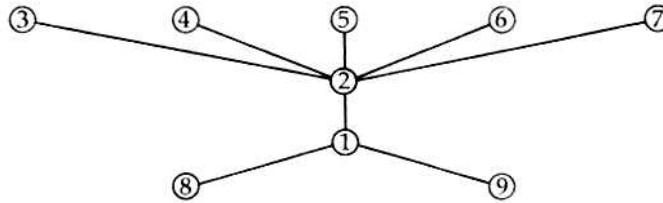
A fourth task is determining how arguments are linked together in longer chains of argumentation. As we observed above in connection with De Morgan's thesis, sometimes the conclusion of one argument can also function as a premiss of a subsequent argument. Hamblin calls the resulting chain a "thread" or "development" of arguments.<sup>10</sup> Geach speaks of *themata* formed by chain-like linkings of individual argument *schemata*.<sup>11</sup> Such procedures were utilized by the medievals in applying syllogistic to extended arguments. Classical first-order logic is indifferent to this phenomenon because classical implication is transitive and no distinction is made between the cases where P implies Q mediately or immediately.

#### IV. ANALYSIS OF ARGUMENTS

Scriven [16] uses a method of circled numerals connected by lines to map

out what one takes to be the structure of an argument in the sense of identifying the conclusions and sorting out which premisses are supposed to support what conclusion, with the help of what other premisses. An example [16, 78f.] will illustrate the gist of this technique.

We can be proud that (America has turned the corner on the depression of the last few years.) At last (the many indexes of recovery are showing optimistic readings.) (The rate of inflation has slowed), (unemployment has more or less stabilized,) (inventories are beginning to drop,) (advance orders are starting to pick up,) and the best news of all-(the average income figures are showing a gain.) (The doomsayers have been discomfited,) and (the free enterprise system once more vindicated.)

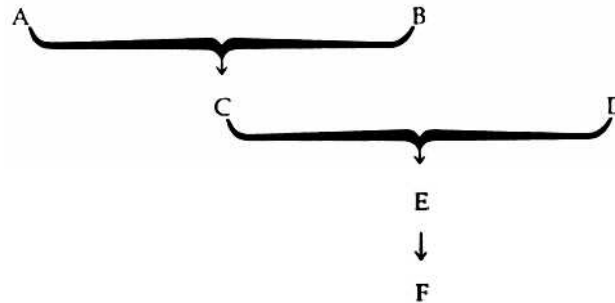


According to the reading given by Scriven, ① is the key conclusion. ③, ④, ⑤, ⑥, and ⑦ are marshalled in support of ①, and summed up in ②. ⑧ and ⑨ are two further assertions drawn from ① but not from each other.

Where "each of the considerations has weight only in conjunction with others" (p. 80), Scriven ties the premisses together with a horizontal bracket (p. 42), e.g.  $\underbrace{\textcircled{1} + \textcircled{2} + \textcircled{3} - \textcircled{4}}_{\text{the + reads "with" the -}}$

"despite"). Geach [4] uses a similar method. If we have a sound argument from A and B to C, and another from C and D to E, and yet another from E to F, then according to Geach, we have a sound argument from A, B and D to F. Geach [4, 66] uses this form of diagram.

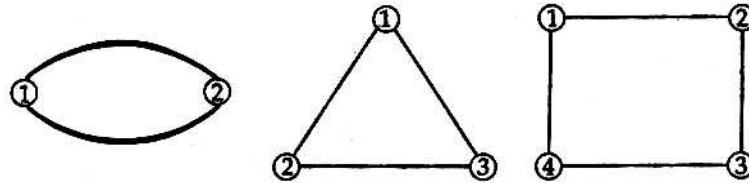




Ehninger [3] uses a similar technique of arrows to join argument chains and clusters.

This general type of method is extremely useful in the practice of evaluating arguments, and helps to provide a working basis for helping with our four tasks of applying logic in the analysis of arguments.<sup>12</sup> It is particularly helpful in enabling us to sort out which are the premisses and conclusions of an extended argument, and in linking arguments up in a chain-like fashion (the fourth task) as indicated in the illustration from Geach given above. However, the use of this method raises many theoretical questions about what we are doing in using it, and I would like to raise some of these questions.

The Scriven-Geach-Ehninger technique can easily be adapted to express cyclic patterns of argument, e.g.

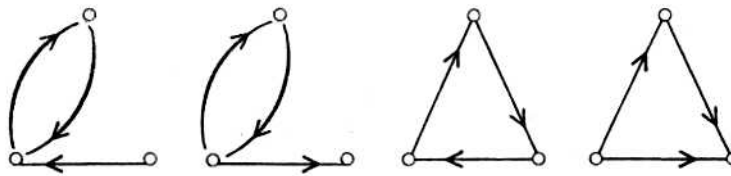




But how are we to understand such cycles? Do they represent the fallacy of *petitio*, or might they sometimes be non-fallacious in nature? Could they possibly be ways of modelling instances of the dependency *petitio*? It seems to depend on what is meant by the points (circled numerals) and steps (lines).

## V. DIGRAPHS OF ARGUMENTS

The various diagrams above are obviously reminiscent of the familiar diagrams of graphs in graph theory - see Harary [7]. The question then is whether an argument analysis of the Scriven-Geach-Ehninger type could be

viewed as a directed graph.<sup>13</sup> A *directed graph* or *digraph* consists of a finite nonempty set  $V$  of *points* (nodes, vertices, etc.) and a set  $X$  of ordered pairs of points. These ordered pairs are called *arcs* (directed lines, edges). Digraphs are usually drawn as points connected by arrows, as in the illustration below representing the digraphs with three points and three arcs.



A *loop* is a line that joins a point to itself, e.g. . If more than one line joins two points, e.g. , it is called *multiple lines*.

An arc of a digraph can be thought of as a binary relation, and as such its properties are relatively weak as relations go. It need not be reflexive—that is, we can have loops or not as we wish. It need not be transitive—just because there is a line from  $v_i$  to  $v_j$  and one from  $v_j$  to  $v_k$ , there need not be a line from  $v_i$  to  $v_k$ . And it is not symmetrical or asymmetrical. In a (non-directed) graph it need not even be non-symmetrical but in a digraph, it is non-symmetrical—that is, if there is a line from  $v_i$  to  $v_j$ , there may or may not be a line from  $v_j$  to  $v_i$ .

The relative non-committal nature of digraph theory accords well with what I take to be the spirit of the Scriven-Geach-Ehninger method in not requiring symmetry or asymmetry. Let us now turn to the key questions of transitivity and loops.

A *walk* of a graph  $G$  is an alternating sequence of points  $v_i \in V$  and lines  $x_i \in X$ ,  $v_0, x_1, \dots, v_{n-1}, x_n, v_n$ , beginning and ending with points, and where each line is incident with the two points immediately preceding and following it (See [7, 13]). In terms of the theory of relations, the notion of a walk permits a kind of transitive closure<sup>14</sup>—if there is a line from  $v_i$  to  $v_j$  and a line from  $v_j$  to  $v_k$ , it does not follow that there is a line (arc) from  $v_i$  to  $v_k$ , but it does follow that there is a walk from  $v_i$  to  $v_k$ . In terms of argument analysis, what this means is that if there is an argument from one premiss-set to a conclusion, and then from that conclusion as premiss to a second conclusion, and so forth from that point to some end statement, then we can say after Hamblin that there is a "thread" or "development" of arguments, as we put it earlier a "chain" of argument, from the initial premisses to the end conclusion. In other words, digraph theory models the structure of our fourth task of argument analysis very nicely. In graph theory we have the distinction between a walk which may have many intervening arcs, and a "single-step" arc  $v_i, v_j$  where there are no points  $v_k$  between  $v_i$  and  $v_j$ . In the theory of argument analysis, we are back to the distinction—tacitly relied on

by De Morgan-between a single-step argument and chain-like collocation of argument steps to produce a complex argument.

Suppose we give in to the temptation to think of an argument in this graph-theoretic way. Each argument is composed of a set of points which represent bundles of statements that are "premisses" or "conclusions." A pair of points is joined by a directed line that represents the step from the initial point (the premisses) to the end point (the conclusions). A walk, or sequence of arcs, represents a thread of argumentation, a sequence of premisses and conclusions joined together to form a longer chain of reasoning. Now some pregnant philosophical questions: what do the points and the arcs really represent in arguments? By focussing attention on the *petitio* these questions may be sharpened.

In a graph theory, a walk is said to be *closed* if  $v_0 = v_n$ , and *open* otherwise. A closed walk with  $n \geq 3$  distinct points is called a *cycle*.<sup>15</sup> For our purposes it is nice to define a *circle* as a walk that is a loop, multiple lines, or a cycle. In the context of argument analysis, a natural doctrine of circular argument may be formulated as follows. A loop represents an equivalence *petitio* and a circle of  $n \geq 2$  represents a dependency *petitio*. In looping, one has argued that P on the basis of P. In cycling where  $n \geq 2$ , one has started a chain of argument with initial premiss P and "arrived back" at final conclusion P. Interestingly, all circles come out to be "purely formal" circles of this sort, just as De Morgan would have liked, but how "strictly logical" a theory of this sort really is depends on how we interpret the points and lines. It would not do, for example, to interpret an arc as a logical implication because implication is transitive. Of course, as far as the theory of directed graphs is concerned, we have virtually limitless freedom in how we wish to interpret points and lines. But the question is rather one of finding practical interpretations that are well suited to the analysis of arguments.

## VI. WHAT'S WRONG WITH ARGUING IN A CIRCLE?

Eberle [2] proposes that we make epistemic logic more realistic by adopting as a binary operator on statements a relation  $I(P, Q)$  to be thought of as representing an actual inference carried out by some inferrer. He then proposes a system, proved sound and complete, that has the following property in place of Hintikka's rationality assumption: if P is known to be true and Q is inferred from P, then Q is also known to be true. This system is the one I would find most applicable in the analysis of actual argumentation because in an argument analysis we are concerned with attempting to determine which inferences an arguer has actually made. The main problem with Eberle's system for this application is that I is transitive: if  $I(P, Q)$  and  $I(Q, R)$  then  $I(P, R)$ . But it seems to me that transitivity is a highly significant sort of rationality assumption in itself. Why should all actual arguers be unerringly transitive in their inferences, any more than they are in their

preferences?

I propose instead that we define  $I_D(P,Q)$  initially as a non-transitive binary relation, the relation of *direct inference* of Q from P, and then define its transitive closure  $I_I(P,Q)$  as follows:  $I_I(P,Q)$  if, and only if, there are statements  $R_1, R_2, \dots, R_i$  such that  $I_D(P,R_1)$  and  $I_D(R_1,R_2)$  and ... and  $I_D(R_i, Q)$ . We read  $I_I(P,Q)$  as the relation of *indirect inference* of Q from P. Now we can think of a graph yielded by an argument analysis as a specimen of an epistemic inference. A "single-step" arc from  $v_i$  to  $v_j$  is a direct inference from the statement (premiss)  $v_i$  to the statement (conclusion)  $v_j$ : A "multi-step" walk from  $v_i$  to  $v_j$  represents an indirect inference from an initial statement (initial premiss) mediated through a sequence of argument steps to an end statement (end conclusion).

Putting inference in this epistemic framework helps us to see what can be wrong or fallacious about cyclic patterns of inference. If we take the basic notion of inference  $I_D$  as above, along with its ancestral relation  $I_I$ , there is nothing demonstrably wrong with circular inferences as actual inferences carried out by some inferer. What can be wrong, as the philosopher told us,<sup>16</sup> is that if there is an epistemic order of propositions, circular reasoning may violate this ordering. What is the right ordering? This question is perhaps the most fundamental one for epistemology, and I will not by any means try to answer it here, but only to point out that there are possible orderings that exclude circles. For example, a linear ordering of the points  $v_i$  excludes circles. A tree-like ordering of the points  $v_i$  excludes circles. And so on. The idea is that we can have different species of *rational inference* as we add different conditions on the basic epistemic digraph which by itself only represents the actual inferences of a reasoner, bereft of various levels of rationality that may be epistemically imposed on his sequence of inferences.

In other words, we are in the pleasant position of being able to concur with Aristotle that *petitio* can be seen as an epistemic fallacy. The lines of an argument digraph can represent the epistemic dependency of the end point on the initial point, and therefore in any sequence of inference through a chain of propositions which represent linkings of premises and conclusions there can be an ordering of the propositions such that each is better known than its successor. Let us say for example that we have a linear ordering of propositions  $v_0, v_1, \dots, v_k$  such that each  $v_i < v_j$  is less well known than  $v_j$ . What this implies is that the chain of inferences at any arbitrary point  $v_i$  can never form an inference with the pair  $\langle v_i, v_j \rangle$  if  $v_i \leq v_j$ . Thus circles of either the dependency or equivalence variety can be ruled out by setting epistemic priorities.

It is worth emphasizing that, so construed, circular inference is not identical with the fault of choosing premisses that fail to be more certain than one's conclusion. Rather, circular inference is one particular species of this more general shortcoming of arguments. It is that species where either Q is inferred from itself, or from some P that is also inferred from Q.

*Footnotes*

\*Research for this paper was supported by a grant from the Social Sciences and Humanities Research Council of Canada.

<sup>1</sup>This phrase is taken from [19].

<sup>2</sup>For an elaboration of these points, see [19].

<sup>3</sup>A detailed account of these two conceptions is given in [19].

<sup>4</sup>For more on this issue, see [20].

<sup>5</sup>The reader should look to [21] for a detailed account of these dialogue-sequences.

<sup>6</sup>( $\beta$ ) represents the inadequacy of the arguer's premisses to convince his opponent by argument. In part VI we will distinguish between the analogous shortcoming in an epistemic context (as opposed to the dialectical context here) and the fallacy of *petitio principii*.

<sup>7</sup>See [21].

<sup>8</sup>See also Mackenzie [13].

<sup>9</sup>See Nicholas Rescher, *Plausible Reasoning*, Assen/Amsterdam, Van Gorcum, 1976.

<sup>10</sup>[5, 229].

<sup>11</sup>[4, 66].

<sup>12</sup>Extended examples of application of this method to the analysis of argumentation are given in Vignaux [17, 294] and Johnson and Blair [10, 177f.].

<sup>13</sup>A useful text on digraph theory is Frank Harary, Robert Z. Norman and Dorwin Cartwright, *Structural Models: An Introduction to the Theory of Directed Graphs*, New York: Wiley, 1965.

<sup>14</sup>Transitive closure of inferences is defined in part VI.

<sup>15</sup>See [7, 13].

<sup>16</sup>In part 1.

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