GAMES, GRAPHS AND CIRCULAR ARGUMENTS

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§ 1. Introduction

The traditional logical fallacy called *petitio principii* (begging the question, arguing in a circle, *circulus probandi,* etc.) as traditionally conceived has been an elusive target for logical analysis. It has never been made clear why ostensibly circular arguments are in some precisely specifiable sense "fallacious". From a point of view of formal logic, we would scarcely want to deny that a proposition is a deductive consequence of itself. However, we hardly want to rule that every equivalence proof in mathematics that argues from A to B, and then back from B to A, has to thereby be fallacious.

It is shown in [11] that one traditional account of the *petitio,* called the equivalence conception, rules that an argument is circular, just in case the conclusion is the same as a premiss. But it has never been made clear what is meant by `sameness', for strict identity would be too narrow a workable criterion, while logical equivalence (in classical first-order logic) would be too wide.

A second traditional account, called in [11] the dependency conception, postulates that an argument is circular if some premiss rests on or depends on the conclusion. The notion of dependency seems to reflect an asymmetrical concept of directionality of proof, in the sense of `starting at A and going to B', which is unfamiliar as an item in the logician's vocabulary. How essential is this notion of `precedence' of propositions for the study of circular argument? Some historical remarks on the traditional conceptions of circular reasoning will set the stage for our argument that `precedence' is essential in examining circles. The complete and careful history of the *petitio* remains to be done, as a topic of importance in its own right. For the present, we want to garner some guidance from traditional accounts that might be helpful in working towards a useful model of circular argument.

Hamblin [3, p. 32] states that the *petitio principii* might be explained by noting that *"principium petere"* is the vulgate translation of Aristotle's original Greek τõ εν αϱχῆ αιτεῖσθαι which means something like, "beg for that which is the question-at-issue." Whence the familiar name, "begging the question." In a disputation of the Greek pattern, if one person proposes an argument to another, she may ask to be granted premisses on which to build. Thus, according to Hamblin [3, p. 33], "[t]he fallacy consists in asking to be granted the question-at-issue, which one has set out to prove."

According to Hamblin [3, p. 74], Aristotle gives two distinct accounts of question-begging. In the *Topics* and the *De Sophisticis Elenchis,* the fallacy is studied as an error of question-and-answer dialogue. However in the *Prior Analytics,* Aristotle is concerned with the formal validity of arguments in scientific proofs.

In the *Prior Analytics* [3, 64b 30] Aristotle writes that demonstration proceeds from what is certain and more prior. Thus one way a demonstration can fall short is when the premisses are not better known to be true than the conclusion to be proved. However if the demonstration goes both ways, from premisses to conclusion and also *vice versa* (as in circular arguments), it cannot possibly succeed as a demonstration. For the premisses cannot be better known than the conclusion, while at the same time the conclusion is better known than the premisses. Thus for Aristotle, circularity of demonstration is a special case of failure of the precedence-relation `A is better known than B (as part of a demonstration)'. More detailed analysis of Aristotle's doctrine of circularity is to be found in [15].

The Aristotelian doctrine of the *petitio* was widely promulgated in the middle ages. For example, in William of Sherwood's Logic [6, p. 158] we find the statement that the acceptability of an inference is inseparable from its producing belief regarding a doubtful matter. This objective can only be accomplished, according to William, on the basis of "prior and better known premisses" [6, p. 158]. William concludes that a circular inference may be formally valid [the consequence proceeds from necessity], yet still fail to be a useful inference because the required relation of precedence for a correct inference is lacking in the circular case.

In modern times, the Aristotelian notion of a formally valid argument has flourished through the development of mathematical logic through Boole, Whitehead and Russell, and beyond. But the notion of precedence in a demonstration has languished. Indeed, textbook treatments of the *petitio* by the nineteenth century had begun to exhibit a marked skepticism about the notion of precedence. Whately [10, p. 179] for example, began to wonder whether priority might be an essentially subjective, or psychological factor.

> It is not possible ... to draw a precise line, *generally*, between this Fallacy [*petitio*] and fair argument; since, that might be fair reasoning, which would be, to another, "begging the question;" inasmuch as, to the one, the Premisses might be more evident than the Conclusion; while, by the other, it would not be admitted, except as a consequence of the admission of the Conclusion.

August DeMorgan, a staunch defender of formal logic, carried this skepticism even further, declaring that an argument is only properly circular if - never mind elusive questions of priority - there is strict identity between the conclusion and one of the propositions that make up the premisses. He lobbied for a strictly formal version of the equivalence conception, ruling ". .. strictly speaking, there is no formal *petitio principii* except when the very proposition to be proved, and not a mere synonyme of it, is assumed." [2, p. 254]. This criterion seems unconscionably narrow however, and Alfred Sidgwick [9, p. 194] was led to remark of it that "such a restriction would be very much at variance with the popular acceptation of the term." The basic problem in modern terms, as outlined in [12], is that by DeMorgan's criterion no argument of the form `$A \vee B$, $\neg A$ therefore B' could ever be circular, because neither premiss is strictly identical to the conclusion. Yet arguments of this form do appear to be intuitively circular in some instances, e.g. where the premiss $A \vee B$ is being concluded on the basis that B is true.

Of course the exponent of DeMorgan's position would reply, as pointed out in [12], that the form of the argument at issue is more perspicuously identified by viewing it as a linkage of two arguments.

$$\dfrac{\dfrac{\begin{array}{c}A \vee B \\ \neg A\end{array}}{B}}{A \vee B}$$

The proposition B is conclusion for the first argument but also premiss for the second. So construed, the argument is circular by DeMorgan's criterion because $A \vee B$ appears both as premiss and conclusion. The general suggestion implicit in this rejoinder is that (single) arguments can be linked together into "chains" of arguments. Thus an argument may be thought of as a sort of sequence of subarguments. What the defender of DeMorgan's position is saying then is that we should accept as a hypothesis that if an argument is truly circular, then its manifest formal circularity will be revealed in a sufficiently long chain of argument-steps. Enough "logical form" will be brought out by putting together enough chain-like linkages of subarguments.

However, the problem with this suggestion. like the precedence idea of Aristotle DeMorgan wished to criticize, is that it is by no means obvious how it could be given a structure in the mainstream of developments in modern mathematical logic, where implication is transitive, and validity is not a property of a whole structure or network of extended argumentation. The investigation of argument circularity suggests asking different questions. If steps P and Q appear in an argument of a logical system, which comes first, P or Q ? In fact, it may be the case that neither precedes the other, but that both are needed to reach a `conclusion', C. Generally, `P precedes Q' is defined to mean that some rule in the logic has P appearing as a premiss and Q as a conclusion. This requirement gives us connections between some steps of the argument and no connections between others, as well as a direction in the sense of `starting at P and going to Q'.

The above approach is especially amenable to the theory of directed graphs, and we introduce this theory in our analysis.(1) Happily, the theory of graphs helps us to clarify our perceptions of circularity, as we shall see in the next section and subsequently. An *argument* is formally considered to be a set of substitution instances of the rules of a logical system. The directed graph of the argument then indicates connections between propositions in the sense that one follows from

(1) Having come to discover the work of Shoesmith and Smiley [8] towards the end of composing this paper, we were pleased to see their use of graph theory to analyse sequences of arguments. Although our approach is fundamentally different in certain respects, both in motivation and structure, we think the two approaches could be profitably compared in subsequent work.

another by the application of a rule, and also indicates direction by means of moves from every premiss to the conclusion of each application of a rule. A circle in an argument then becomes a *cycle* [4] in the corresponding directed graph, that is a set of propositions A_1, . . . A_n such that A_1 is connected to A_2, A_2 to A_3 and so on, and also A_n is connected to A_1, with arrows from A_1 to A_2, A_2 to A_3, and finally A_n to A_1.

Our use of these new structures for the analysis of arguments raises many questions about features of arguments that have often been studied more at the pragmatic level, particularly in the recent concerns on argumentation in linguistics. For example, it seems that some contexts of argumentation require a strong notion of precedence while others have no requirement of precedence at all. These traditions are not unfamiliar in philosophy as well. Coherence theorists, for example, may view circularity as quite benign whereas a "cumulative" context of argument as described in [13] may suggest a rigid exclusion of circular argument. The suggestion then is that different argumentative contexts strongly affect how we view circularity as fallacious or permissible. We analyse these contextual profiles by games of dialogue.

Hence we will follow the suggestion that *petitio* can be explored by a game-theoretic approach, harking back to the disputational view of `begging the question' in Aristotle's Topics and *De Sophisticis Elenchis*. Hamblin [3, p. 268f.] constructs a question-and-answer game in which two participants put forward statements (Statement A), questions (Why A?), and take on or retract commitments to propositions. According to Hamblin, adding the following two rules to the game blocks *petitio: (W)* `Why A?' may not be asked unless A is a commitment of the hearer and not of the speaker; (R1) The answer to `Why A?', if it is not `Statement A' or `No commitment A,' must be by way of statements that are already commitments of both speaker and hearer. To see how these rules jointly block *petitio,* consider the following sequence of a Hamblin game.

WHITE	BLACK
(1) Why A?	Statements B, $B \supset A$
(2) Why B ?	Statements A, $A \supset B$

When Black replies `B, $B \supset A$' at (1), (R1) ensures that both propositions B and $B \supset A$ are commitments of both participants. So White's query `Why B ?' at (2) is disallowed by (W).

The Hamblin game enables us to see how circles can be expressed and regulated in a game-theoretic structure, but a problem pointed out in [13] is that a circular sequence can be constructed in a Hamblin game even with (W) and (R1). Consider the following sequence, where White is initially committed to $A \supset B$ and $B \supset A$, and Black is committed to A, B, $A \supset B$, $B \supset A$, and C.

WHITE	BLACK
(1) Why A?	Statement B, $B \supset A$
(2) Statement A	Statement C
(3) No commitment B ; why B ?	Statements A, $A \supset B$
(4) Statement B	

Black uses B to prove A, but then when White retracts his commitment to B, Black uses A to prove B. Is there a circle? It seems hard to say definitively, but according to an interpretation of the above sequence given in [13], a case can be made out that the sequence constitutes a *petitio*.

It seems therefore that logical dialogue-games are connected to the *petitio principii* in important ways, but that so far in the literature, the analysis of the *petitio* has not been given in precise enough form to make this inter-connection clear or fruitful. We hope to show that our graph-theoretical model of circular argument can be connected to the study of logical dialogue-games in a helpful way.

§ 2. Some Technical Definitions and Examples

In this section we introduce the technical language of graph theory and set the stage for examining arguments in this context. Although we shall not be developing here an in-depth theory of digraphs as they relate to logical arguments and games, it seems wise to at least establish a firm base from which this could be done at some later date.

We work with a set of atoms P and n_i-ary operators $\Delta = \{\Delta_i\}_{i \in I}$ where I is an index set and the n_i's are positive integers.

A well-formed formula or *wff* is defined recursively as follows.

1. Each atom is a wff.
2. If A_1, A_2, ... A_n are wffs and Δ is an n-ary operator, then $\Delta(A_1, A_2, \ldots A_n)$ is a wff.

Intuitively, when one argues for a particular proposition P, say, one presents a set of premisses, and tries to demonstrate how `logically' one can get to P from these premisses. The validity of the argument can be rigorously checked by comparing each step in it with the set of rules allowed. We formalize these ideas in the following definitions.

An *argument* is a non-empty finite set of wffs with one distinguished from the others. Notation: $A = \{A_1, A_2, \ldots; A_{n+1}\}$, where A_{n+1} is the wff distinguished from the others. A_{n+1} is meant to play the role of the *conclusion* and we shall call it that. The other wffs take the role of the *premisses.* The technical term *set* implies that no two of the A_i, $1 \leq i \leq n+1$, are the same. We therefore avoid the most blatant occurrence of *petitio.*

A *formal system is* a triple $F = (P, \Delta, R)$ where P is a set of atoms, Δ a set of n_i-ary operations and R a set of arguments called *rules.*

We shall use small roman letters (e.g. p, q, ...) with or without subscripts to denote atoms, and capital roman letters (e.g. A, R, .. .) with or without subscripts to denote wffs. In general, script capitals denote arguments or rules or sets of atoms, arguments or rules.

Any wff A can be considered as a function $A(\{p_i\})$ of the distinct atoms p_i occurring in it. Hence we define a *substitution instance* of a wff $A(\{p_i\})$ to be a wff $A(\{q_i\})$ where q_i replaces p, for each i, such that if $p_i = p_j$ then $q_i = q_j$. A *substitution instance of a set of wffs* $\{A_i\}$ is a set $\{B_i\}$ where each B_i is a substitution instance of A_i, and such that wherever p is replaced by q in some element of the first set, it is replaced by q in each element of the first set.

A *demonstration* of an argument $A = \{A_1, \ldots A_n; A_{n+1}\}$ is a finite set of substitution instances of elements of the rules R such that

a. A_{n+1} appears as the conclusion of precisely one substitution instance, while the conclusion of every other substitution instance occurs as a premiss of at least one substitution instance.

b. Each premiss of a substitution instance is either a premiss of *A* or the conclusion of precisely one other substitution instance; and no premiss of *A* appears in the conclusion of a substitution instance.

The conditions (a) and (b) here are intended to provide a valid deduction of A_{n+1} with $A_1, \ldots A_n$ as the starting point, while also indicating a `direction' in the sense that the goal is A_{n+1} and indicating that other substitution instances of the rules, however valid, may not be included if they are extraneous to the goal of arriving at A_{n+1}. It may in any event be the case that not all of $A_1, \ldots A_n$ are needed in a demonstration of A.

If the wff A of the argument A appears in a substitution instance of a demonstration of A, then A is said to be an *initial premiss* of the demonstration. Wffs of substitution instances of the demonstration that are not in A are called *implicit premisses*.

We now turn to the graph-theoretic representation of a formal system and its demonstrations. We need first of all some basic terminology from the theory of digraphs. (See [4] for example.)

A *digraph* is a tuple $D = (V, \mathscr{A})$ where V is a nonempty set of elements called *vertices* and $\mathscr{A}$ a family of ordered pairs of elements of V, called *arcs*.

A *diwalk* of a digraph from vertex v to vertex w is a finite sequence of distinct arcs $(v_0, v_1), (v_1, v_2), \ldots (v_n, v_{n+1})$ where $v_0 = v$ and $V_{n+1} = W$.

A *dipath* from v to w is a diwalk in which $v_i \neq v_j$ if $i \neq j$.

A *dicycle* from v to w is a dipath except that v = w.

A digraph $D' = (V', \mathscr{A}')$ is a *subgraph* of the digraph $D = (V, \mathscr{A})$ if $V \subseteq V'$ and $\mathscr{A} \subseteq \mathscr{A}'$.

We proceed to describe the construction of a digraph corresponding to the set of wffs $\{A_1, \ldots A_n\}$.

Define

$V_1 = \{A_1, \ldots A_n\}$

$V_2 = V_1 \cup \{B_i^1\}$ where each is the conclusion of a substitution instance of a rule of the form $\{V_1^1, V_2^1, \ldots V_{n_1}^1; B_i^1\}$, $V_k^1 \in V_1$, $1 \leq k \leq n_1$.

In general, $V_j = V_{j-1} \cup \{B_i^{j-1}\}$ where each B_i^{j-1} is the conclusion of a substitution instance of a rule of the form $\{V_1^{j-1}, V_2^{j-1}, \ldots V_{n_{j-1}}^{j-1}; B_i^{j-1}\}$, $V_k^{j-1} \in V_{j-1}$, $1 \leq k \leq n_{j-1}$.

Define $V = \bigcup_j V_j$.

Define $\mathscr{A} = \{(V_k^j, B_i^j) \mid V_k^j, B_i^j$ as above, $1 \leq i, j; 1 \leq k \leq n_j\}$.

The *digraph* of $A = \{A_1, \ldots A_n\}$ is then $D(A) = (V, \mathscr{A})$. We represent it diagrammatically as in figure 1 below.

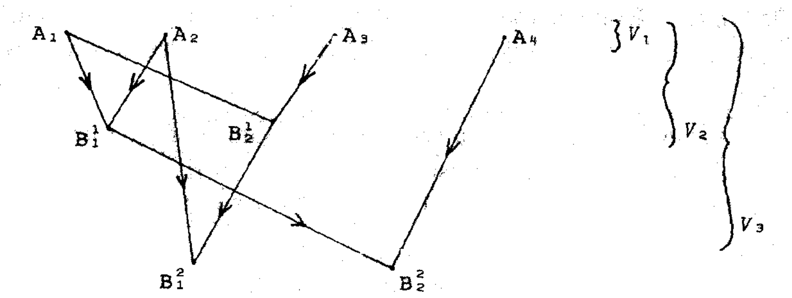

Figure 1

Clearly, the sets V and $\mathscr{A}$ need not be finite.

The ordered pairs (V^j_k, B^j_i) which are the arcs in our digraph represent the `precedence' in which we are so interested. Since $B^j_i$ `follows from' V^j_i we denote this in the digraph by an arrow from V^j_i to B^j_i placed on the line representing the arc (V^j_i, B^j_i).

We shall make the following convention. If $A = \{A_1, \ldots A_n; A_{n+1}\}$ is an argument, then $D(A)$ means the digraph $D(\{A_1, \ldots A_n\})$. This gives us immediately the following result.

Theorem. An argument A has a demonstration if and only if the conclusion of A is an element of the vertex set of D(A).

It should be clear that the set of substitution instances constituting a demonstration of an argument A forms a subgraph of D(A).

Thus for a particular argument $A = \{A_1 \ldots A_n; A_{n+1}\}$, the digraph D(A) provides us with a diagram of how the rules are applied, what demonstrations exist, which premisses are not needed in finding a demonstration, and so forth. We follow several conventions in drawing such digraphs, $(V, \mathscr{A})$:

1. Distinct elements of V are represented by distinct points of the diagram.

2. If $(A, B) \in \mathscr{A}$ then the points of the diagram corresponding to A and B are joined in the diagram by a line with an arrow on it pointing from A to B. If A appears as a premiss and B as a conclusion of a substitution instance of the rule R_α, we label the arrow with the letter or number α.

Example 1. Examine the digraph in figure 2 of the argument A = {A_1, A_2, A_3, A_4, A_5, A_6; A_7} in a given system. Demonstrations are given by

1. {A_6; A_7} where this is a substitution instance of the rule R_5.

II. {A_1, A_2, A_3; B_1}, a substitution instance of R_3, and {A_5; B_2} a substitution instance of R_5, and {B_1, B_2;A_7} a substitution instance of R_1.

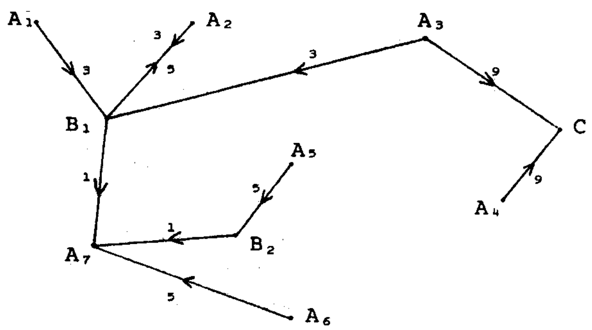

Figure 2.

The digraph also indicates that {A_3, A_4; C} and {B_1; A_2} are substitution instances of R_9 and R_5 respectively. Also, A_4 is not used in any demonstration of A_7.

Example 1 is a very `formal' example of how a digraph works, in the sense that it has been removed from any contextual interpretation for the wffs, - arguments and rules involved. For our second example, we use a more `natural' setting of a dialogue as a back-drop to the digraph schema.

Example 2. In the dialogue below, there is an attempt to justify induction as a rational procedure. More than occasionally it has been argued that attempts to justify induction cannot escape being based on induction themselves, and that consequently any attempt to justify induction is a *petitio.* The problem with the serious examination of such real-life allegations of *petitio* tends to be that it requires engagement into a protracted sequence of argumentation with many premisses and conclusions, steps of reasoning, and objections and replies. It has been conjectured that the longer and more complex the sequence of steps in the argument, the more effective is the *petitio* as a strategem.

Despite such complications, we feel it is very useful to study a bit of moderately realistic argumentation where there is an interesting allegation of *petitio* that can be precisely traced out. Consequently we present below a dialogue that we have constructed ourselves but that is realistic enough to provide an interesting study specimen of a *petitio* allegation. The topic of the dialogue is the justification of induction.

BLACK: I think that part of the problem about justifying induction as a reasonable procedure is that the problem itself is never stated very clearly·

WHITE: I agree. But I think Hume went to the heart of the matter formulating the problem as a question of how reasonable our expectation is that the future will resemble the past. I mean, the sun has risen in the past, in all our past experience, so is it reasonable to expect that it will continue to rise in the future?

BLACK: Well I know that you, White, have always been skeptical about the possibility of a rational justification, whereas I have taken the opposite point of view. So let's postulate the issue of our disagreement as a proposition. Let's say that the proposition I am required to prove in order to rebut your skepticism is precisely this: our expectation that the future will resemble the past is reasonable.

WHITE: All right. How are you going to prove it then?

BLACK: Well, first I suppose we should have to define some terms. What does `reasonable' mean for example? Whatever we might mean by this term, would you agree at any rate that if an expectation works, that is if circumstances turn out in such a way as to justify it, then it may be said to be reasonable? That is, do you agree to this proposition: any expectation that works is reasonable.

WHITE: Well, all right. That seems a harmless enough assumption.

BLACK: Moreover, you are willing to agree that our past expectations have worked.

WHITE: Well, by and large, yes. That is not something I care to dispute.

BLACK: You also concede that a past expectation is an expectation ?

WHITE: Well, yes-that is trivial enough.

BLACK: Then you agree that our past expectations are reasonable ?

WHITE: Well yes, I'd have to. In fact that follows deductively from everything I have conceded so far. But what I'm not sure of is whether our expectations about the future are reasonable. How can you prove that?

BLACK: Well, could you agree to this proposition: we can expect our future expectations to work?

WHITE: No, of course not. That's just the sort of thing I'm skeptical about. Maybe our expectations will at some future point start not working.

BLACK: Well, of course. But right now we can expect our future expectations to work, can't we?

WHITE: I suppose so, but ....

BLACK: Well, just agree to it tentatively as a provisional hypothesis.

WHITE: If you like.

BLACK: Now (Black begins to take on a cunning look, and wrinkles his brow in concentration) : we already agreed that any expectation that works is reasonable, and we tentatively accept the proposition that we can expect our future expectations to work. I now propose that it follows deductively that we can expect our future expectations to be reasonable.

WHITE: If you like to draw that inference, yes. It appears to be valid.

BLACK: Given that deduction, I now propose another, namely

Our past expectations are reasonable.
We can expect our future expectations to be reasonable.

Our expectation that the future will resemble the past is reasonable.

Thus the proposition we originally set out to prove is indeed provable, and induction can be justified.

WHITE: Now I think I see what you are up to. I do not dispute your arguments, and indeed am quite willing to agree that they are all deductively valid. But let's go back a bit to a previous stage of the argument, if you will.

BLACK: Of course.

WHITE: The part of your proof that is questionable is that premiss to the effect that we can expect our future expectations to work. You wanted me to agree to it. I, of course, did not agree, but when you tried to get me to agree, you used reasoning something like this. You conceded that, at some point in the future our expectations might become unreliable, but that right now we can expect them to work. Wasn't that your argument?

BLACK: Yes, as I recall. Don't forget that you accepted the premiss that our past expectations have worked.

WHITE: Yes, that is precisely the point I wish to make. On the basis of your utilizing that premiss, how else could you be proving to me that the disputed premiss is true except by assuming that our expectation that the future will resemble the past is reasonable. In other words, this inference must be part of your agreement.

Our expectation that the future will resemble the past is reasonable.
Our past expectations have worked.

We can expect our future expectations to work.

BLACK: Well, it does seem to be perfectly valid.

WHITE: But that is not my dispute with it. Rather my criticism is this. The major premiss of this small argument above is the very proposition you set out to prove in the first place. You are begging the question-you assume the very proposition you are supposed to prove!

BLACK: (Looks a little uncomfortable.) Hold on a bit. I mean, from my point of view, it is reasonable to expect that the future will resemble the past. That is something I accept.

WHITE: Yes, of course, but you are supposed to be proving it to me. And since I am dubious about the proposition that our future expectations will continue to be reliable, I will of

> course likewise be dubious that the future will resemble the past in point of reliability of expectations. You've got to give me some independent evidence that either of these propositions is true or you're simply trapped in your own cycle of assumptions.

Here the dialogue ends. A simple analysis of some of the structure of the demonstration conveyed by the dialogue can be given by identifying the following six propositions.

p: Any expectation that works is reasonable.
q: Our past expectations have worked.
r: Our past expectations are reasonable.
s: We can expect our future expectations to work.
t: We can expect our future expectations to be reasonable.
u: Our expectation that the future will resemble the past is reasonable.

Now the main outline of Black's argumentation can be represented as below. White's first move along with Black's subsequent argument and statement make clear at the outset that the ultimate conclusion to be proven by Black is u. The first step in the argument runs as follows, letting Ex = x is an expectation, Wx = x works, Rx = x is reasonable, Px = x is in the past, and Fx = x is in the future.

p $(\forall x)\ [(Ex \wedge Wx) \supset Rx]$
q $(\forall x)\ [(Px \wedge Ex) \supset Wx]$

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r $(\forall x)\ [Px \wedge Ex) \supset Rx]$

Formal logic and Black's query at his fifth move suggest that q could trivially be re-written as its equivalent: $(\forall x)\ [(Px \wedge Ex) \supset (Ex \wedge Wx)]$. Thus the argument is valid in classical logic, so far, as White concedes. Next, Black proceeds to argue.

p $(\forall x)\ [(Ex \wedge Wx) \supset Rx]$
s We can expect that $\forall x)\ [(Ex \wedge Fx) \supset Wx]$

---

t We can expect that $\forall x)\ [(Ex \wedge Fx) \supset Rx]$

This form of argument is classically valid, except for the `We can expect that' prefix, for which there is no known formalization. At any rate, White does not dispute the formal correctness of the argument, only wanting to indicate his lack of commitment to s.

Now Black proposes his third argument. Let Rxy stand for the relation `x resembles y in a certain respect' (a symmetrical relation).

r	$(\forall x)\,[(Ex \wedge Px) \supset Rx]$
t	We can expect that $(\forall x)\,[(Ex \wedge Fx) \supset Rx]$
u	Our expectation is reasonable that $(\forall x)\,(\forall y)\,[(Fx \wedge Py) \supset Rxy]$

To clean up this argument, Black needs at least one further premiss, e.g. $(\forall x)\,(\forall y)\,[(Rx \wedge Ry) \supset Rxy]$.

Now having seen how each of the three separate parts of Black's argument can be worked up into a valid argument, where the validity is not disputed by White, let us retrace White's objection to what has transpired.

White questioned the premiss s. Indeed White responds that Black's justification for s takes the form of argument, `u, q, therefore s'. Black however points out that this argument should not be disputed for its validity. White agrees, but still has an objection. This objection is not to any one of the above four arguments *per se,* at least as far as the individual validity of each is concerned. Rather, his objection becomes clearer if you put all four together at once. White's objection is that the digraph of the whole sequence of Black's arguments contains a cycle (s, t, u).

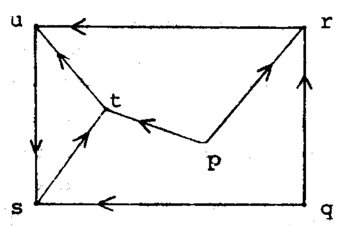


Figure 3

Here then is the real form of the argument from a point of view of White's allegation of *petitio principii*, displaying the controversial cycle. Let us now proceed to the job of stating conditions permitting us to fairly rule whether the circle is vicious or fallacious.

§ 3. Inevitability

You could say that arguing in a circle is wrong because the question should never be begged. That is, if the respondent asks "Why p?" the opponent should never be able to use p as a premiss in any reply as justificatory response to that question. What the respondent wants is some *other* proposition than p that implies p. But then one can always ask: is this requirement of the respondent a reasonable one? What is wrong if the opponent replies "p because q" to the respondent's query "Why p?" then "q because p" to the query "Why q?".

Probably the most natural response is this. When the respondent asks "Why p?" it means that he does not accept p, and he wants some q that implies p that is a proposition he *does* accept. By these lights, appeal to p is out of place because it is clear at the outset that p is a proposition he does not accept. This response is implicit in Hamblin's rule (W) of § 1.

But the natural response itself begs the question, because we often do ask "Why p?" even when it is not clear that the asker does not accept p. Even though I firmly believe p, I might still quite legitimately ask you "Why p?" in order to see whether you might have grounds for accepting p. Thus we can ask: why should the respondent be restricted to querying propositions he does not accept? And why should he have the right to demand only propositions in response that he does accept? Should not the opponent have the right to "try out" responses that the respondent may or may not accept?

The natural response might persist that the object of the game is to change the belief of the respondent. That is, to get him to accept something he did not previously accept. But again we may ask: why is that the only legitimate sort of dialectical game? What is wrong with a game where the objective is not that but something else-say, for the respondent to find new arguments for something he already might believe, or at least not clearly reject?

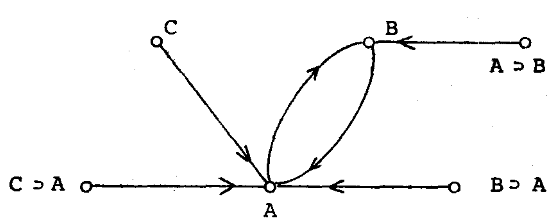
So the natural response does not tell us what is wrong with arguing in a circle. It merely tells us that any game of a certain sort with circles is wrong, but does not tell us why a game of this sort is the only one that is permissible. In a word, it begs or at least postpones the question.

Of course, as we saw in § 1, the Hamblin game even with (W) and (R1) does not entirely block circular arguments in the game. Perhaps circles would be prohibited altogether if retractions of commitments were not allowed. Even so, we must insist that the point remains-how can we be assured that (W) and (R1) are reasonable restrictions? If we have a long chain of arguments, why must each step in the chain be a commitment of both questioner and answerer at that step? The restriction appears arbitrary and does not allow the questioner reasonable latitude to construct sequences of arguments with premisses other than definite commitments of both parties.

Thus as it stands, the natural response is not good enough to tell us precisely when a circular argument is in some sense wrong. It still remains quite possible that in some types of dialectical games, circular patterns of argument may be quite acceptable. For example, in some games the objective of argument may be to look at different possible sequences of arguments in a hypothetical way, e.g. an equivalence proof in mathematics. Here the circle could be revealing, and need not by itself be evidence of some logical error or inadvertency of proof.

Moreover, there are instances where arguments are in some sense circular, or at least contain cycles, but appear nevertheless to be fairly benign rather than vicious. Consider the following dialogue and matching graph.

	WHITE	BLACK
(1)	Why A?	Statement B, B ⊃ A
(2)	Why B ?	Statement A, A ⊃ B
(3)	Why A ?	Statement C, C ⊃ A



There is a cycle in the graph of this argument, but it is a relatively benign one. True, at (2) White argued in a circle by utilizing A as a premiss. But when queried again at (3), White "broke out of" the circle by providing an evidentiary support extrinsic to the cycle {A, B}. We conclude that although a cycle may appear in the graph of an argument, it does not follow that the argument as a whole should be considered altogether fallacious.

True, Black by his circular detour forced White to repeat a question, but if that is an error, or breach of rational argument, it does not 'seem to constitute what we fully mean by petitio. Quite to the contrary, Black's argument overall is a reasonable one, unless it can be shown otherwise.

It seems then that no circle is inherently bad of itself. What is being called into question is rather the validity or justification of a *proposition* in a particular system. It seems natural to argue that if every available evidential route to the conclusion of an argument lies on a cycle, then that conclusion is not justifiable independently of itself in that system. We thus label such a conclusion *inevitably circular,* and every argument that `seems' to justify it, *fallaciously circular.* The example of figure 4 illustrates what we have in mind. No matter which way an argument for A is given, it falls on a cycle also including A. There is no `way out' available for the proponent of A to take.

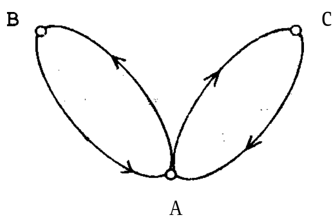


Figure 4.

Before detailing a more complicated example, we introduce some terminology.

If, in a digraph *D(A)* there is a finite diwalk from the wff P to the wff Q, $P \neq Q$, we write $P \to Q$.

We say Q is P-free if $P \not\to Q$.

Example 3. Let *D(F)* be as in figure 5 for some formal system *F* where A, B, . . . S, T are wffs, and suppose that each arrow represents a different rule.

Notice that K → J while there is an argument for J which does not depend on K: {I; J} is valid, while K ↛ I. However, every argument for K depends on J.

If every argument for a wff A depends on a wff B, we write B ↠ A.

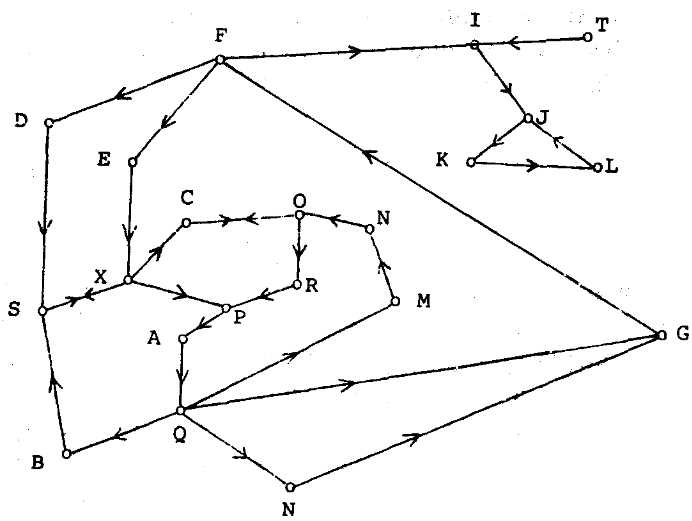


Figure 5

It is also the case that P ↠ Q and Q ↠ P. More interestinglv. anv argument for P involves P, so that P is in fact inevitably circular. The same is true for Q.

In the dialogue on induction, the conclusion originally set to be proven by Black is u (Our expectation that the future will resemble the past is reasonable). However, towards the latter stages of the argument, White points out that Black has utilized previously as a premiss the proposition q (Our past expectations have worked). Moreover, Black now wants White to accept the conclusion s (We can expect our future expectations to work). Then White proceeds to argue by questioning as follows. How else could you (Black) be proving to me that s is true except by making the assumption of u, along with q, as a premiss? The result of this choice of premisses is clearly evident in the graph, where (s, t, u) is a cycle. Black is undone by this argument-he appears to be caught in an inextricable cycle.

However, the fact is that u is not inevitably circular. For although r infers u, neither t nor s infers r. Whatever the fault of Black's argument then, it is not that there is an inevitable circle for the conclusion u. Not every point accessible to u is on an inevitable circle leading back to u. So while White has made a reasonable allegation of *petitio* against Black by citing a circle (s, t, u), he has not shown that a fallacy has been committed on the hypothesis that inevitable circularity is the fallacious kind of circularity.

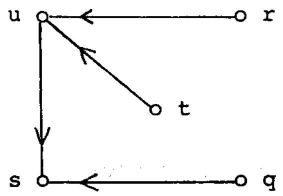
§ 4. *Precedence*

In the induction dialogue, White concedes that u and q taken together imply s. Black of course reasons that White will accept s, once it is pointed out to him that s is implied by premisses u and q. White however reacts quite unexpectedly. He isn't inclined to accept s. In fact, once he sees that s follows deductively from u and q, he is inclined to search for one of this pair that he needs to reject as well, in order to maintain consistency. He quickly realizes that he is disinclined to accept u, and indeed that he has been so disinclined all along - in fact u is the original proposition Black was supposed to prove. So White accuses Black of *petitio*.

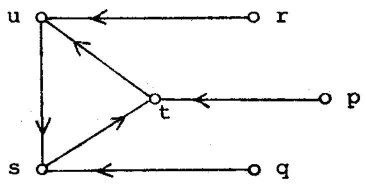
Here then we are back to the notion of direction in an argument. If A implies B, an acceptance of A may serve to convince an arguer to accept B. But if the disinclination to accept B is even greater than the inclination to accept A, the direction of the argument may be the other way around. The participant in argument may reject A as a consequence of his adamant rejection of B.

Perhaps it is this directionality that lies at the basis of White's objection stated in his final response. That is, when there is a directed arc of a graph from p to q, the participant to whom the argument is directed requires or expects that p is better established or carries greater evidential weight than q. Thus the arrow going from p to q signifies that the inference should go in this direction, not the other way around - the weight of evidence of p should be enough, along with another premiss, to direct the argument recipient towards a greater acceptance of q on the basis of p.

By these lights, the force of White's final objection is as follows. u and q together imply s, as you, Black, have correctly argued. Nonetheless, I am disinclined to accept u, and of course have been from the outset. But let's take it a step further. Look back over your previous sequence of arguments. Why should I have to accept u, on the basis of these arguments? Well, because of r and t.



But the problem with that argument is that I am also disinclined to accept t (We can expect our future expectations to be reasonable) because, as I see it, t is no better established by the evidence than u. I still need better established premisses. I ask: Why t? But according to your previous argument, t is based on the premisses s and p.



Hence the problem. t rests on s. But remember that s rested on u, and u rested on t! There is a cycle, and the cycle never permits us to use any one of {s, t, u} as an independent evidentiary basis for the other two propositions. Thus as White says, Black is "trapped in his own cycle of assumptions".

What appears to be the basis of White's objection as portrayed in the paragraph above is not that u is inevitably circular - and indeed it is not - but that, because it is a cycle, some relation of evidential priority between the pairs of the members of the cycle seems to be "cancelled out" by the cycle.

We are back then to the fundamental Aristotelian notion of precedence of propositions in an argument. For any pair of propositions A and B, if A is prior to B in an argument, then B cannot be prior to A. Reason: if A is prior to B, then the direction of inference is from A to B in the sense that A may be used as a premiss that will, by deductive argument, make B better established than it was before the argument.

The assumption is that the evidentiary wellknownness of A, in order to make A of utility as a premiss, must be prior to that of B. Once the deduction is granted however, the value of B should be adjusted upwards to a plausibility value equal to (and not greater than) A. Once A has been so utilized as a premiss for B however, B could never be used as a premiss in an argument that has A as conclusion. Reason: to be useful as a premiss, the value of B must be greater than that of A. But as was just shown above, the value of B should not be greater than that of A, if A has been used as a premiss for B in a previous deduction. Thus arguing in a circle, from A to B, and then subsequently from B to A, violates some requirement of evidential priority.

The Aristotelian approach of the last paragraph suggests that we adopt certain postulates governing the operations of relations of precedence. First, attach to each of the set of propositions in an argument a number representing the relative plausibility(2) of each proposition agreed upon by the participants in a disputation. In the graph of the argument, each proposition at each vertex of the graph will have a number assigned to it. Then the first condition, according to the Aristotelian notion of a demonstration, must require that every proposition in an argument (demonstration) must be such that every proposition that "precedes" it as a premiss, or functions as a premiss for some previous conclusion that leads to that proposition, must have a greater plausibility than the proposition in question. We can state this condition as follows. Recalling that $p \rightarrow q$ represents the fact that there is a directed walk from p to q on the graph.

(C1) $(\forall p)(\forall q)$ (If $p \rightarrow q$ then $\text{plaus}(p) > \text{plaus}(q)$)

This condition requires that the plausibility values of the vertices on

(2) The conception of plausibility adopted here is that of Rescher [7].

any graph must be so ordered that as we go along any directed walk we go from greater to lesser values.

This condition would effectively ban circles in the sense that you could not, following (C1), consistently assign plausibility values at all on a graph where there is a cycle. If you had $p \rightarrow g$ and $q \rightarrow p$, then $p \rightarrow q$ requires plaus(p) > plaus(q), which contradicts the assignment required by (C1) of plaus(q) > plaus(p) to $q \rightarrow p$. However, as we will see below, (C1) is a very strong condition.

Another condition suggested by the foregoing discussion is the basic rule of plausible inference given by Rescher [7]. This rule stipulates that the plausibility value of the conclusion of a plausible inference should be at least as great as the least plausible premiss (least plausible premiss rule). This suggests the following general condition.

(C2) $(\forall p)\ (\forall q)$ **(If** $p \rightarrow q$ **then plaus(q)** $\geq$ **plaus (p))**

Clearly however (C1) is too strong if taken in conjunction with (C2), for these two conditions, as formulated above, are inconsistent with each other!

An underlying problem with (C1) is that in many arguments it does not allow a disputant enough latitude in seeking out sequences of argument that might eventually lead to more plausible premisses. In argument, (C1) demands more plausible premisses immediately, rather than giving a participant in argument "room to argue". True, (C1) does represent a model of evidential increment in which the less well known propositions are always based on the better known. But many longer sequences of arguments are in practice, perfectly reasonable, even if they do not meet this very strong requirement.

One way out would be to relax (C1) to the weaker requirement,

(C1A) $(\forall p)\ (\forall q)$ **(If** $p \rightarrow q$ **then plaus(p)** $\geq$ **plaus(q))**

While (C1 A) seems reasonable perhaps, by itself, if conjoined to (C2), it requires that all plausibility values for all points of a graph be equal. Not a very promising approach.

The question of circularity, we should note, is very much influenced by which set of conditions we decide to adopt. By (C1), all circles are interfered with. While circles may appear in the graph of an argument, consistent sets of plausibility-values cannot be attached to a set of points on a circle. By (C2), circles are quite permissible. Indeed, by (C2) all the points on a circle are given the same value. This might seem significant except for the fact that, taken with (C1 A), it yields the result that any point on the graph will take the same value.

The upshot is that we should be encouraged to explore some weaker alternatives to (C1) in addition to (CIA). We need to ask: in an argument, what should a questioner be asking for in querying an opponent's proposition?

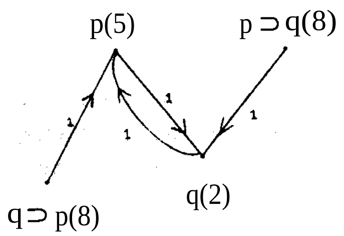
In a dialectical situation, when a respondent asks the question "Why p?" intending to ask for his opponents justification for p, he wants her to produce some proposition q, which may be of two types: (1) q may be a proposition that is more plausible than p, and that implies p, or (2) if not this, then q should at least be a proposition that could be implied by some other proposition that is more plausible than p. The idea is that the opponent should be allowed a number of question-answer justification steps to produce some plausible basis for p. She is not required to do it in one step-more latitude may be allowed. But at some stage, she must produce a q that is more plausible than p in the series of implications.

The opponent wins if she comes up with the appropriate sequence of arguments and premisses. If not, after an agreed upon number of finite moves, she loses.

Let us grade the propositions in this game for plausibility-value on a scale from 0 to 10 (min --> max). Now if asked "Why p?" where p has value 5, there is nothing wrong as such with replying "q and $q \supset p$" where q has a value of only 2. The respondent simply asks "Why q?" in the hope of getting some r with plausibility plaus(r) > 5. But the sequence must end at some point, we presume.

Now what if the opponent though, responds "p" to "Why q?" and adds that $p \supset q$ has value 8, and R_1 says that p and $p \supset q$ imply q. Let s say we have the following situation.

2. Why q? Opponent's reply #2



1. Why p? Opponent's reply #1

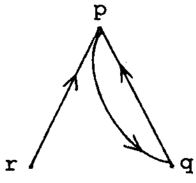
Taken as a single response, there is nothing wrong with the opponent's second response either. In fact it is a good one because it would raise the plausibility value of q from 2 to 5.

But of course as a quest for a more plausible basis for p, the dialogue has not gone anywhere, and the respondent has no choice but to reiterate his original question "Why p?" What is wrong with this? Well, perhaps, nothing too serious yet except the following:

(1) The opponent has forced the respondent to repeat a question-the very same question, that is.
(2) One hopes that the opponent is not going to go around the cycle again and again, *ad infinitum*.
(3) The opponent has delayed the respondent's quest for some r of greater plausibility than p that implies q. It is a kind of filibuster.

What is wrong is merely that p remains r-free, for some r, plaus(r) > plaus(p). True, it is merely a failure of the opponent to yet provide a worthwhile evidential basis. But it is a special kind of evidential failure, for a cycle on which p occurs can never raise the value of p any higher than it starts out.

If a dialogue-sequence goes in a cycle, this does not mean that it is "wrong" as a trial run, but once it has been run, we can see that it has been a waste of time. As soon as non-circular evidential justification is forthcoming, the cycle is redundant. But if no adequate non-circular basis is found, the cycle is of no use in working towards terminating the dialogue. So suppose the opponent's responses to "Why p?" have taken this sort of pattern, where plaus(r) > plaus(p), but plaus(q) < plaus(p).



Respondent	Opponent
1. Why p?	q
2. Why q?	p
3. Why p?	r

The thing is that the "r" response is overruling- the "q" response does not settle anything. For the arrow from p to q boosts the value of q up to that of p. But q is thereby limited- it can't go any higher than p, so it can't be part of an argument that raises the value of p any higher.

Thus in logical games of dialogue generally, we are not entitled to rule that the respondent loses the game or commits a fallacious move if she argued in a circle. She may need the latitude to search for new premisses, and that latitude of argument may sometimes result in cycles. A cycle does not preclude the respondent's going on to produce a plausible argument not requiring the premisses on the cycle. In such a case, other than being weak strategy for the respondent herself, there may not be anything wrong *per se* with a demonstration that contains cycles.

However, in the special game of dialogue where we adopt (Cl) as the rule to assign plausibility values to the graph of the argument, a circular demonstration cannot be assigned values. In this special (Aristotelian) game of plausible demonstration, circular arguments are never allowed, and may be viewed as incorrect precisely because they violate (C1). But for logical games of dialogue in general, (C1) would be an unduly restrictive requirement because it may not allow a participant enough latitude to construct sequences of arguments that may eventually, but not immediately, link up to premisses of greater plausibility.

We conclude that whether circular argumentation is or is not fallacious in a specific instance depends on the requirements for the assignments of plausibility values in the particular game of dialogue one is supposedly engaged in. What remains is to further evaluate the available plausibility assignment rules for different games.

§ 5. *Conclusions*

Condition (C1) required that for any proposition you care to choose, p, every proposition in the set A of all arguments for p, had to be of greater plausibility value than p. That is, the following strict inequality was postulated.

$$(1) \quad \mathrm{plaus}(p) < \min_{q \in A \in A_p} \{\mathrm{plaus}(q)\}$$

We found that while this condition was characteristic of a certain sort of plausibility game, it was too strong to reasonably characterize all types of games of dialogue. We also mentioned a weaker requirement,

$$(2) \quad \text{plaus}(p) \leq \min_{q \in A \in A_p} \{\text{plaus}(q)\}$$

This approach would allow a participant more latitude than (1) in assigning plausibilities, but would still exert a certain measure of control over the argument by requiring that participant to discount propositions in his argument that are less plausible than the proposition at issue. He needs premisses that are at least as wellfounded as the conclusion he is supposed to establish. This approach would allow circular arguments, and even allow propositions of unequal plausibility value to be on the same circle in an argument.

Which of these conditions is the better one to use in the evaluation of allegations of circular argumentation? To answer this question we have to turn to some general considerations on logical games of dialogue. We begin with some basic notions following Hintikka [5] as the appropriate framework.

A game of dialogue consists of two participants α and β, in the simplest case. Each participant must prove a thesis, A_0 and B_0 respectively, in order to win the game. The "move" of each participant consists in the advancing of one proposition, each player taking a turn. After i moves, α will have a set of propositions $A_0, A_1, \ldots, A_i$ and after j moves, β will have a set of propositions $B_0, B_1, \ldots, B_j$. For each player his or her moves will be listed as a column in a two-column tableau. Hintikka [5] then adds a number of other features to this basic structure- rules regulating the forms of propositions, rules defining what constitutes a "proof' of one proposition from others, and rules concerning the asking and answering of questions. The first player to "prove" his or her thesis from given premisses (initial propositions) (3) – that is, in the least number of moves - wins the game. One of the purposes of these formal games is to model argumentation and traditional *sophistici elenchi.*

As with more familiar games, it is important in games of dialogue to design rules that are restrictive and yet permissive enough to generate interesting competitions. We need to exclude moves that enable one participant to easily win by trivial ploys. For example, we must forbid either opponent's filibustering by allowing him or her infinite repetitions that trivially block the opponent from winning. Some types of

(3) These premises include the initial premisses plus subsequent concessions.

rules that control strategies are local in the sense that they pertain to single moves or pairs of moves, e.g. the rule defining what counts as implication on the propositions. (1) and (2) are global rules of dialogue, since they apply to all arguments over the whole sequence of propositions in the dialogue.

A third approach would be to require that at least one premiss in every argument be of greater plausibility value than the proposition at issue, i.e.

$$(3)\quad \forall A \in A_p \ \text{plaus}(p) < \max_{q \in A} \{\text{plaus}(q)\}$$

Such a condition appears to be of no particular significance to us however. The fact that a high plausibility proposition, e.g. a tautology, appears somewhere in a network of argumentation is not by itself decisive in evaluating the over-all worth of the argument. The weaker possibility

$$(4)\quad \forall A \in A_p \ \text{plaus}(p) \leq \max_{q \in A} \{\text{plaus}(q)\}$$

appears likewise to be of little interest in itself. We might note however that both these approaches freely allow circular arguments, being similar in this regard to (2).

The problem then is that applications to games of dialogue would suggest that while (1) is too restrictive, (2), (3), and (4) appear so permissive as to be uninteresting. An alternative approach is suggested by the emphasis of Rescher [7] in selecting out the least plausible premiss in organizing plausibility screening of propositions. According to the framework of argument analysis advanced in previous sections of this paper, we look at all possible demonstrations from a given set of initial premisses. Putting these two approaches together, we want to evaluate argumentation as follows. Look at all the arguments and in each one, select out the least plausible proposition - its "weakest link". Then amongst these arguments, pick the one that has the greatest plausibility proposition for its weakest link. That is, we select the least plausible proposition in each argument, and then of these propositions select the most plausible. This amounts to the following condition.

$$(5)\quad \text{plaus}(p) = \max_{A \in A_p} \{\min_{q \in A} \{\text{plaus}(q)\}\}$$

The understanding here is that we consider the plausibility of both the initial and implicit premisses in each argument. Returning to the example of figure 3, we see that

$$\text{plaus}(u) = \min \{\text{plaus}(p), \text{plaus}(q), \text{plaus}(r), \text{plaus}(s), \text{plaus}(t)\}$$
$$= \text{plaus}(s) = \text{plaus}(t).$$

Hence all points on the circle must be assigned the same plausibility. This happens because, in a sense, the argument is 'closed'. Everything follows from p and q, and there is no alternate route to any of u, s, t.

Suppose we break this `closed' system by introducing a proposition v such that $v \rightarrow u$. (see figure 6).

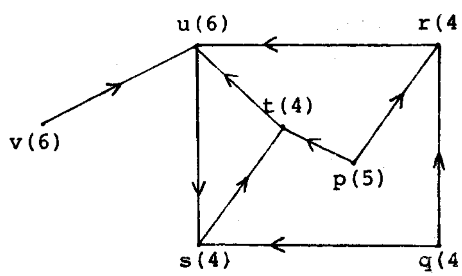


Figure 6.

Assign v a plausibility larger than that of p and q. Then automatically, plaus(u) = plaus(v), while s and t still depend on p or q and hence have smaller plausibilities than plaus(u).

Thus, in the closed system, the plausibilities on a circle are the same, indicating that the circle is indeed `vicious'. If the system is not closed, it is possible to have different plausibilities on the circle, indicating that it is not vicious at all.

The reasonableness of the assumption of condition 5 is therefore enhanced by the fact that with it, it seems possible to differentiate between fallacious and non-fallacious circles.

Now let us get back to the question of whether circular arguments are in any sense fallacious. To sharpen the question let us assume that the game of dialogue is a *dispute* in Hintikka's sense that A_0 is the negation of B_0. In our earlier example of dialogue on the justification of induction, Black and White may not be completely opposed. Probably both want to see whether induction can be justified by rational argument. However, let's assume that they dispute the question and that Black is supposed to prove that induction can be justified.

White made the objection that Black was "trapped in the cycle of his own assumptions". Does White have a legitimate and serious grievance? If (1) is the appropriate rule for the plausibility weighting of Black's argumentation then White's allegation of *petitio* is reasonable, at least up to a point. For, as our analysis of the induction dialogue showed, Black has indeed argued in a circle. However, the circle was not an inevitable one.

We therefore adjudicate as follows. The dialogue should not be terminated in White's favour at this point. Rather, Black should be given the option to see if by further moves he could provide additional arguments extrinsic to the existing cycle. If further dialogue resulted in an inevitable circle, White could be in a better position to mount a criticism of *petitio.*

Furthermore, the cycle may be assigned plausibility values for its propositions provided that the global rule for the assignment of these values is one of the rules (2) - (5). In these cases, there need be nothing wrong *per se* with a circle in Black's arguments. However, if the rule (1) is supposed to be the appropriate rule for this particular dialogue, then White's criticism shows that Black's argument fails to meet the standard for plausible reasoning.

We rule therefore that Black and White should agree which of conventions (1) to (5) applies to the game of dialogue they are engaged in. Insofar as we have argued that (5) is the appropriate rule for optimal two-person plausibility games of dialogue, we also rule in this instance that White's criticism of *petitio* does not constitute a refutation of Black's argument.

Put bluntly and somewhat paradoxically, what we are saying is that there is really nothing wrong (very often) with arguing in a circle. Put somewhat more carefully, we are saying that *petitio principii* is a more difficult fallacy to fairly and justifiably "nail down" as a conclusive refutation of an opponent's argument than is often presumed. In order to prosecute such a charge, several conditions must be met, and our sample dialogue, insofar as it is a relatively typical allegation of *petitio,* reveals that many such allegations are based on presumptions containing *lacunae.*

We are prepared to conclude then that, according to our investigations, the following cautionary remarks about the *petitio* should be made. If α and β are in disputation, and β has argued from a set of initial premisses to his thesis B_0 in such a way that cycles occur in the graph of his argument, α does not thereby automatically have a good case for a refutation of β's arguments on grounds of circularity. Indeed where one of (2) - (5) is the agreed-upon principle to assign plausibility values, α has no grievance, or sound allegation of `fallacy' at all. Perhaps the cycles in β's arguments may indicate "inelegancies" or "detours" on the part of β's strategy of demonstration, perhaps not. But they do not represent a strategem that permits β to win the game by any trivial or deceptive sequence of moves.

If the appropriate principle is (1) however, we do concede that circles in β's arguments would violate the Aristotelian requirement that the propositions in β's arguments should be ordered strictly in assigning plausibility values. In such a case, the allegation of *petitio* has bite. However, we think that although (1) has its place, the more generally characteristic rule for games of disputation is (5).

In order to fairly evaluate many commonplace criticisms of argumentation, including the *petitio,* often a good deal of reconstruction and questioning of the participants needs to take place in order to clarify the nature of the dispute. The premisses, rules, theses of the participants - all those factors we have defined - first need to be more fully agreed upon by the participants before an evaluation can take place. Once all this is done however, we think we have shown why many allegations of circularity are harder to defend and less worrisome than the Standard Treatment of the textbooks would have us believe.

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