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PRAGMATIC INFERENCES ABOUT ACTIONS *

Does it matter whether natural conversations about human actions and consequences of actions ever conform to formal principles of logical inference? After all, was Hume not justified in his skeptical position that contingent matters of fact do not admit of necessary connections? What more needs to be said about the extent to which actions may be said to be logical?

One problem stems from a long-standing member of the informal traditional fallacies, the so-called circumstantial *ad hominem* fallacy. Suppose Dr. Smith delivers a small lecture to patient Jones on the adverse effects of smoking, citing statistical and other results to prove to Jones that several dangerous diseases. such as cancer of the lungs and throat, are directly caused by smoking. Dr. Smith concludes that one should not smoke, and makes it clear that what she means to say is that Jones should stop smoking. Suppose additionally that during the course of this conversation Dr. Smith is herself smoking a cigarette, blowing out plumes of smoke to punctuate her points. The tradition is that Dr. Smith's performance may be open to an allegation of circumstantial *ad horninem* by Jones' reply, "How can you say that one should not smoke while you yourself smoke? Isn't that inconsistent?" The problem: Is Jones' criticism a reasonable one?

This problem is compounded by the initial difficulty that we need to sort out how much of what Smith has put forward is to count as Smith's argument. For example, perhaps the medical and statistical evidence cited by Smith for the conclusion that smoking is harmful to health is good evidence. If this much of the argument is acceptable to Jones, then Jones is certainly incorrect (and perhaps commits a form of *ad hominem* or *ad feminan*) to reject it simply because the immediate source of it herself smokes.

But is that all there is to the argument? Surely the ultimate conclusion of Dr. Smith's argument, 'One should not smoke,' is a normative rather than purely statistical or medical conclusion. This normative conclusion contains a directive to a certain course of action. It is not therefore fair to count as part of the argument Dr. Smith's own personal advocacy of

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that conclusion, in particular her own performance of smoking while propounding the argument that one should not smoke? The question turns on whether the performance of an action can count as part of the argument. We seem to be at the pragmatic edge, if not right in the pragmatic wastebasket.

Here, then, is the disputed question. Some critics would say that there is no logical inconsistency in condemning smoking while one smokes. Hence they would say that the *ad hominem* allegation is trivial or pointless. Critics would recognize the legitimacy of a deontic-action-theoretic sort of inconsistency in the conjunction of "Everybody ought not to smoke" with the present act of the speaker's smoking. To the extent to which one's actions may conflict with one's directives to certain types of action, one's position as advocate of an argument should be open to serious *ad hominem* criticism, according to this viewpoint.

Which side of this disputed question is more defensible depends on the extent to which actions may be said to be logical. So it does matter very much for our understanding of how arguments are criticized whether conversations about actions conform to principles of logical inference.

A primary objective of our work will be to study conditionals of the form 'If an agent brings something about then she also brings something else about'. Such conditionals are important to study for many reasons, but we will particularly look to see how they function in what Goldman (1970) has called level-generation of actions. We want to confirm Goldman's thesis that more than one kind of conditional is involved, and show that these conditionals have similar, but by no means identical, properties. The logical structures appropriate to the two conditionals primarily studied turn out to be applications, respectively, of the relatedness logics and dependence logics of Epstein (1979, 1980). That is not too surprising in the first case, because relatedness logic was originally constructed to model conditionals in act-sequences, as shown in Walton (1979). However, the reader will see that some significant details of this original modelling have been changed in that part of the analysis given below that pertains to relatedness.

1. ANALYSIS OF ACT-SEQUENCES

According to Goldman (1970, ch. 2) the relationship that obtains between pairs of act-tokens, and that allows act-sequences to be linked together into act-trees, is called *level-generation.* Four categories of level-generation are distinguished: (1) causal generation, (2) conventional generation, (3) simple generation, (4) augmentation generation. Conventional generation is characterized by the existence of rules, conventions, or social practices that link one action to another in a conditional, e.g., If John extends his arm out the car window then John signals a turn. In simple generation, circumstances combine with one action to ensure the performance of another action, e.g., If John dangles a line in the water then John is fishing.

Our concern here will not be directly on rules or background circumstances. We will concentrate on the basic categories (1) and (4) as types of conditionals in act-sequences, and leave the application of our analysis of (2) and (3) to the reader. We will call our version of (1) external generation, and our version of (4) internal generation of actions.

External generation of actions has often been called the accordion effect. Consider this familiar sequence; Smith's moving his finger, Smith's flipping the switch, Smith's turning on the light, Smith's warning a prowler. Each step in the sequence is an action, and some would say the whole sequence is also an action. As has often been remarked on, we can begin with a sequence of natural states of the world - the finger movement, the switch-flipping, the light going on, the prowler's being warned - and the element of human agency is transmitted over that sequence by its initial appearance at the first step. Like the other kinds of act-sequences studied by Goldman, this one falls naturally into an order as given above. But this sequence is ampliative and extrinsic-directed, stretched out by nature into different space-time regions, unlike the internalized and intrinsic-directed sequence of progressive containments of (4). The rationale of an external sequence would seem to be that as each step in the description of the complex of events is taken, a new element yet one that is still somehow related to the previous step is introduced. That is, each event is related to an event that is immediately proximate to it in the sequence in such a fashion that the separate links fuse together to make up a chain-like sequence. Each member stands in such a relationship of having to do with the activation of its immediate neighbours that an orderly sequence of action is possible over the complex of individual actions thus ordered. How each stage of the sequence is made to happen has something to do with how each other stage is made to happen. But what does "has something to do with" mean in this context?

Before we can answer this question, we have to ask what is related by this relation in an act-sequence, i.e., what is an action? The first presumption of the present theory is that an action-sentence can be restated (clarified, analysed) in such a way that there is a proposition expressed that is said to be made true by the agent. For example, "Sue puts the pencil on the desk" must be analysable by some form of paraphrase like the following: "The pencil is on the desk", a proposition made true by Sue. This form of analysis was originally due to St. Anselm - see Henry (1967, pp. 120ff.) [1] - and has recently been developed by Pörn (1977) and Walton (1979).

As Davidson (1967) pointed out, it is a nontrivial task to see how commonplace action sentences could be made to conform to such a paraphrase. For example, "Cass walked to the station" cannot be precisely equated with either "Cass made it true that she is at the station" or "Cass made it true that she walked to the station". One way of confronting this problem is to introduce a class of propositions called "action propositions" by Walton (1980) or "act-relations" by Pörn (1977) which meet this requirement: necessarily (p if, and only if, p is made true), and p is possibly true.[2] Since we may presume that it is possible that Cass has two legs even though she has in no way made this true, "Cass has two legs" is not an action proposition. Whereas if Cass made it true that she walked to the station, then "Cass walked to the station" is an action proposition. So in general we presume such a paraphrase of action propositions is a feasible project. That part of the project having to do with propositional calculus will comprise the subject of this paper.

The expressive capacity of the language of actions we construct is limited to the bringing about of changes or leaving the status quo in a given situation. For example, if the door is in fact closed, my options as agent are limited to (a) bringing it about that it is open, or (b) letting it be closed. We will not try to express the idea of the agent letting the door be open or making it closed, if in fact the door is already in a closed state. Thus the language we develop here is that of a special kind of action where the agent is alone in a situation, and can effect a change in that situation or allow the situation to remain as it is.

The more general accounts of action utilizing the 'bringing-about' notion, those of St. Anselm in Henry (1967), and those of Pörn (1977) and Walton (1979), all make a distinction between an agent making it false that p and not making it true that p. The account given here simply does not have the expressive capacity to straightforwardly make the distinction. In other words, the present account is based on a strong assumption of asymmetry between active making false and passive omission.

The language developed here is best applicable to the cases where the agent deliberates on whether or not to act so as to change an already given situation. For example, suppose I am deliberating on whether to dig a hole in my backyard, and the present situation is that there is no hole there. On the present account, I can make it true that there is a hole or let it remain the case that there is no hole. I can't make it false that there is a hole there, or let it be true that there is a hole there. Our language is therefore one of action relative to a given situation.

2. EXTERNAL SEQUENCES OF ACTIONS

By the foregoing type of analysis, we can say that the external act-sequence breaks down into a set of four propositions, each made true by Smith: Smith's finger was moved, the switch was flipped, the light was turned on, the prowler was warned. Accordingly, each stage in the sequence is a proposition, and accordingly we now re-ask our question above as follows: what does it mean to say that each proposition "has something to do with" each other proposition in this set? Readers of Walton (1979) know that the answer is supplied by introducing a relatedness relation $\mathscr{R}$ that is reflexive and symmetrical, but not transitive: $\mathscr{R}(p, q)$ means approximate spatio-temporal coincidence of p and q in the sense that p can affect q or q can affect p. Then p_0 is indirectly related to p_{n+1} if, and only if, there are propositions $p_1, p_2, \ldots, p_{n-1}, p_n$ such that p_0 is related to p_1, p_1 is related to $p_2, \ldots,$ p_{n-1} is related to p_n, and p_n is related to p_{n-1}. Propositions connected in a sequence by intervening related propositions are themselves indirectly related.

The specific context of deliberation about action we propose to model is that of an agent in a given situation confronted by a set of propositions. The situation is divided by the agent into a number of space-time zones or locations. Each atomic proposition takes on a subset of these sectors, assigned to it by the agent. The set of locations of a proposition is called its *zone* or *aegis*. If p_i and p_j share some locations in common between their respective zones, we say that p_i and p_j are related, meaning that making one true can affect making the other true. Thus the structure of how complex propositions are made true or allowed to be false is mediated through the spatio-temporal locations assigned to these propositions by the agent of change in a situation.

We must be careful here to make a clear distinction between atomic propositions and the complex propositions formed by connectives from atomic propositions. Using capital letters, A, B, C, . . . , for complex propositions, our problem is to know how to assign sets of locations to these, based on our way of assigning sets of locations to atomic propositions, p_1, p_2, ..., p_n. We have to be careful to make this distinction clearly. For example, relatedness could be transitive over the atomic propositions, but we will see that it can never be transitive over complex propositions.

We must also be careful that what is meant by a spatio-temporal zone may be complex in certain instances. For example, as Cresswell (1979) notes, 'it is raining in Wellington' does not mean rain is present in all points in Wellington but only in some acceptable subset. For the present however, we overlook these subtleties of language.

We define a proposition as taking on precisely one of two values. We interpret $p = T$ as meaning that what some agent does makes p true. We interpret $p = F$ as meaning that what some agent does lets p be false. In constructing complex propositions, we need to take into account both the "truth-values" and the question of how the propositions are related to each other.[3] For any complex proposition "A and B", we can say that A is made true and B is made true if, and only if, both A and B are made true. Similarly for negation, relatedness does not seem to matter, so we can define $\neg A$ in the classical way too. A is made true if, and only if, it is not the case that A is allowed to be false. And A is allowed to be false if, and only if, A is not made to be true. But it is when we come to define the conditional that relatedness makes a difference.

An external act-sequence is sometimes a series of conditionals. For example, we may presume it is being claimed that "If it is made true that the switch is flipped, then it is made true that the light is turned on" is true for the sequence given above. Not that anyone need claim that it is logically necessary that if A (the switch is flipped) is made true then B (the light is turned on) is made true. But rather we do sometimes presume in certain act-situations that one claims as a matter of fact, or hindsight, that it is not the case both that A is made true and B is allowed to he false by the agent. As Segerberg (1982) points out, however, 'do' is ambiguous. Sometimes, 'the agent does something' implies that success was certain in what he did, other times not. Here we mean to imply that when B is made true by making A true, then it is not the case that A is made true and B is allowed to be false. However, in such a claim more than this is presumed: we also in the case of an act-sequence require that A has something to do with B, *i.e.*, A and B are related. Hence we define $A \rightarrow B$ (If A then B) nonclassically as follows: $A \rightarrow B$ iff and $\neg(A \wedge \neg B)$ (A, B). We interpret $A \rightarrow B$ as meaning: whatever makes A true also makes B true. This definition was in certain respects inspired by the conditional defined in van Fraassen (1969, p. 485), but there are important differences between van Fraassen's conditional and those analysed here.[4]

We now must decide how the complex conditionals are related. When, in general, is C related to $A \rightarrow B$? Consider "If the switch is flipped then the light is turned on." It seems reasonable to think that this conditional is related to "The bulb is working", and also to "The switch is working." Thus the best requirement is this: C is related to $A \rightarrow B$ if, and only if, C is related to A or C is related to B. We can now see why relatedness on complex propositions can never be transitive. Since A is always related to $A \rightarrow B$ and $A \rightarrow B$ is always related to B, it would follow by transitivity of $\mathscr{R}$ that A is always related to B. By such a proposal, every complex proposition would be related to every other one.

Finally, we have to decide whether to define disjunction in such a way as to require relatedness or not. We reason that $A \vee B$ is not made to be true if A alone is made to be true and A has nothing to do with B. Thus we define disjunction as follows. $A \vee B$ is made to be true if, and only if, at least one of A or B is made to be true and A is related to B.

Now let us look to see which forms of inference are valid or not, for the requirements set out above are enough to determine a class of propositional calculi. The tautologies for the requirements above turn out to he precisely those of System $\mathscr{S}$ of Epstein (1979). A decision procedure in the form of modified truth-tables is available, requiring that we take relatedness into account as well as truth-values.

First, *modus ponens* is valid: if A is made true, then if whatever makes A true makes B true, then B is made true. For example, suppose that it is made true that the switch is flipped. Then if whatever makes it true that the switch is flipped makes it true that the light is on, then it is made true that the light is on. The following truth-table demonstrates validity.

A	B	$\mathcal{R}(A, B)$	A	$\rightarrow$	$[(A \rightarrow B)$	$\rightarrow$	$B]$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	T	F	T	F
T	F	F	T	T	F	T	F
F	T	T	F	T	T	T	T
F	T	F	F	T	F	T	T
F	F	T	F	T	T	F	F
F	F	F	F	T	F	F	F

The truth-tables are similar to classical logic except that we have to take into account all possible relatedness relations on the basic propositions, as well as all possible combinations of truth-values.

Readers of Epstein (1979) will know that the System $\mathcal{S}$ that results from our requirements above is a subsystem of classical PC. Listed below are some tautologies that are shared with classical logic.

Some Tautologies of System $\mathcal{S}$ of Relatedness Logic

(1) $A \rightarrow [(A \rightarrow B) \rightarrow B]$
(2) $\neg B \rightarrow [(A \rightarrow B) \rightarrow \neg A]$
(3) $(A \rightarrow B) \rightarrow [(A \rightarrow \neg B) \rightarrow \neg A]$
(4) $A \rightarrow \neg\neg A$
(5) $\neg(A \wedge \neg A)$
(6) $A \rightarrow [\neg B \rightarrow \neg(A \rightarrow B)]$
(7) $\neg A \rightarrow [(A \vee B) \rightarrow B]$
(8) $(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$
(9) $(A \wedge B) \rightarrow A$
(10) $A \rightarrow A$
(11) $A \vee \neg A$
(12) $A \rightarrow (A \vee \neg A)$
(13) $(A \rightarrow B) \rightarrow \neg(A \wedge \neg B)$
(14) $[(A \wedge \neg A) \rightarrow B] \rightarrow [B \rightarrow (A \rightarrow B)]$
(15) $[(A \wedge \neg A) \rightarrow B] \rightarrow [\neg A \rightarrow (A \rightarrow B)]$
(16) $(A \rightarrow B) \rightarrow [(A \wedge C) \rightarrow B]$
(17) $(\neg A \vee \neg B) \rightarrow \neg(A \wedge B)$
(18) $(A \rightarrow B) \rightarrow (\neg A \vee B)$
(19) $(\neg A \vee B) \rightarrow (A \rightarrow B)$
(20) $(\neg A \wedge \neg B) \rightarrow \neg(A \vee B)$

Perhaps some of these theses are worth special comments. Let us take an example of (2), *modus tollens*. If the light is allowed to be off then if whatever makes the switch flipped makes the light go on, then the switch is allowed to be not flipped. This inference is reasonable, for assume that the switch is not allowed to be not flipped. Then it must follow either that it's not made true that whatever makes the switch flipped makes the light go on or that its's not made true that the light is allowed to be off.

Contraposition, (8) also seems reasonable. Assume that whatever makes the switch flipped makes the light go on. Then whatever allows the light to be off also allows the switch to be not flipped.[4a] Reflection should indicate that the remaining inferences are also correct for external act-sequences.

Perhaps even more interesting is that many inferences we would take not to be applicable to conditionals in act-sequences, must fail on the present interpretation. Consider (1) from the set given below. Whatever makes A true need not make it true that whatever makes B true makes A true. Reason: B need have nothing to do with what makes A true. Similarly with (8): whatever makes A true need not make it true that A or B.

Some Tautologies of Classical PC That Fail in $\mathscr{S}$

(1) $A \to (B \to A)$
(2) $\neg A \to (A \to B)$
(3) $\neg(A \to B) \to (B \to A)$
(4) $(A \wedge B) \to (A \to B)$
(5) $\neg(A \to B) \to (A \wedge \neg B)$
(6) $A \to (B \vee \neg B)$
(7) $(A \wedge \neg A) \to B$
(8) $A \to (A \vee B)$
(9) $\neg B \to [\neg A \to (A \to B)]$
(10) $[(B \wedge C) \to A] \to [(B \to A) \vee (C \to A)]$
(11) $[A \to (B \to C)] \to [B \to (A \to C)]$
(12) $[(A \wedge B) \to C] \to [A \to (B \to C)]$
(13) $[A \to (B \to C)] \to [(A \wedge B) \to C]$
(14) $\neg(A \vee B) \to (\neg A \wedge \neg B)$
(15) $(A \to B) \to [(C \to A) \to (C \to B)]$
(16) $(A \wedge \neg B) \to \neg(A \to B)$
(17) $[A \to (B \wedge C)] \to [(A \to B) \wedge (A \to C)]$
(18) $(A \to B) \to [(B \to C) \to (A \to C)]$
(19) $(A \to B) \to [A \to (C \to B)]$
(20) $\neg(A \wedge B) \to (\neg A \vee \neg B)$

Failure of transitivity might be taken to be an unfavourable characteristic for the present analysis of extrinsic act-sequences. For if the first pair of conditionals below is made true, then surely the third is thereby made true.

(K) If the switch is flipped, the light is turned on.
(L) If the light is turned on, the prowler is warned.
(M) If the switch is flipped, the prowler is warned.

Our reasoning here is that the third conditional need not be true in the sense that making it true that the switch is flipped is directly related to making it true that the prowler is warned. Still, the latter pair of propositions is indirectly related, and thus in that limited sense of $\to$, (M) does follow (indirectly) from (K). But we cannot allow unrestricted transitivity, for the reason that remote consequences of actions may not be regarded as part of the act-sequence at all, either directly or indirectly.

Let us assume for example that it is not the case both that the switch was flipped and that the prowler's brother-in-law tripped over his lawn-mower and broke his leg two weeks later. Still, we need not say that by making it true that the switch was flipped, Smith made it true that the prowler's brother-in-law broke his leg. We might, but it would depend on how or whether these two situations were related.

Although transitivity does not generally obtain, we should notice that a close analogy to transitivity may be added to the set of tautologies for $\mathcal{S}$, namely the following schema:

(21) $[(A \to B) \wedge (B \to C) \wedge A] \to C.$

We can show this as follows. In order for (21) not to hold, C must have the value F and each of the three conjuncts of the antecedent must have the value T. So A must have the value T, and therefore for $A \to B$ to take the value T, B must take the value T as well. But these values, namely $B = T$ and $C = F$, make the remaining conjunct, $B \to C$, take the value F.

So because of these truth-values, (21) is a relatedness tautology as well as a tautology of classical logic. So if we have (K) and (L) as above, with 'The switch is flipped' as a premiss, then the conclusion 'The prowler is warned' follows deductively. However, the implication here is reasonable as long as we realize that the last pair of statements quoted above do not have to be directly related to each other. Perhaps we should add parenthetically that (21) would not be a relatedness tautology if we defined conjunction so as to require relatedness, as shown possible by Epstein (1979). But that is not a view of conjunction we want or need to advocate for act-sequences.[5]

Several of the last list of schemata, like (1) and (2), are ones that have often been rejected as "paradoxical", but (10), even though highly paradoxical even in classical logic in its usual truth-theoretic interpretation, does not appear to have been considered as troublesome, by comparison to (1), (2), or (3).

Consider an example. If I flip both switch A and switch B then the light will go on. Therefore, at least one of the following is the case: if I flip A the light will go on, or if I flip B the light will go on. This inference would fail if both switches together were required to make the light go on. So it seems the schema should not be generally true for internal act-sequences. Yet it is easily seen that it is a classical tautology, i.e., that $[(p \wedge q) \supset r] \supset [(p \supset r) \vee (q \supset r)]$ is a tautology of classical PC.

An example will illustrate how deeply problematic this is for deductive implication. Consider any deductively valid argument where both premisses are required.

> Pierre is taller than Quetzil.
> Quetzil is taller than Rudolf.
> Therefore, Pierre is taller than Rudolf.

That is, if either premiss is deleted, the remaining one does not by itself deductively imply the conclusion. The problem is: if you accept the schema above as a principle of classical inference, then you have to accept that if the above argument is valid then at least one of the premisses must by itself deductively imply the conclusion. That is, given the validity of the above argument, it follows that at least one of these arguments must be valid.

Pierre is taller than Quetzil. Therefore Pierre is taller than Rudolf.	Quetzil is taller than Rudolf. Therefore Pierre is taller than Rudolf.

But this is indeed sophismatical. For surely the first argument is deductively valid, but neither of the remaining pair is by itself a deductively valid argument. What is wrong? A good question, for students of propositional calculi, but suffice it to say here that any theory of act-sequences based on the classical logic of propositions has to confront the question of how to deal with this sort of inference.

Pörn (1977) bases his theory of actions on classical propositional calculus. He introduces a modal operator 'it is necessary for something the agent does that *p*' where *p* is a proposition that obeys the laws of classical PC. In line with the present discussion of conditionals, let us turn around Pörn's basic expression to an equivalent 'something the agent does is sufficient for *p*', and confront the following problem. Suppose an agent constructs a machine in such a fashion that if both switches *A* and *B* are flipped then light *C* will go on. But suppose he constructs the machine so that both switches are required to be on for the light to be on. That is, if either is off, the light will not be on. Using Pörn's language, we say that something he does, namely constructing the machine, is sufficient for the following to obtain: if *A* and *B* are both on then *C* is on. By the logic of Pörn's operator we must deductively infer that the following also obtains: it is either the case that if *A* is on *C* is on, or it is the case that if *B* is on *C* is on.[6] But does it really follow?

Our very description of the agent's construction of his machine assures us that the premiss is true but the conclusion is false. For it is not the case that if *A* is on *C* is on, and it is not the case that if *B* is on *C* is on. So it would seem that any analysis of 'if ... then' based on classical logic would have to deal with this dubious implication.

3. INTERNAL SEQUENCES OF ACTIONS

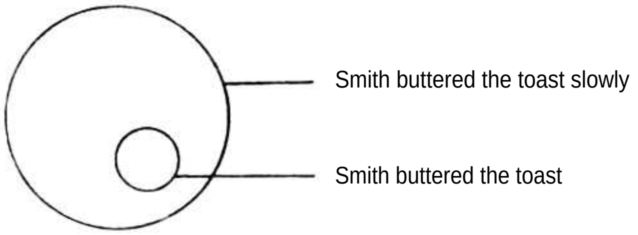
Some stages in an act-sequence are related to other stages as we shall now say, internally. Consider this familiar sequence: Buttering the toast, Smith's buttering the toast, Smith's buttering the toast slowly. Smith's buttering the toast slowly and deliberately, Smith's buttering the toast slowly and deliberately with a knife, Smith's buttering the toast slowly and deliberately with a knife in the bathroom, Smith's buttering the toast slowly and deliberately with a knife in the bathroom at midnight. This sequence falls naturally into the order in which it is given above. The rationale of the sequence would seem to be that as each step in the description of the complex of events is taken, more information about the development of what happened is given. In each subsequent case, the development of the sequence as described is *more specific* as an unfolding situation. That is, there is a sense in which each stage of the unfolding sequence is included in each other stage that comes after it.

We start with the idea that each of the descriptions above contains some items of information telling us how something is made true. The first description of the event contains the information that it was a buttering and that the toast is what was buttered. The second description adds to the information in the first by telling us that the aforesaid toast-buttering is Smith's. It brings forward the previous information, but at the same time adds a new element. The transformation from the first to the second step is one of increased specificity of information about how something was made true. The information contained in the first step is included in the information contained in the second step. The converse does not obtain. That is, information content inclusion in internal event sequences is not a symmetrical relation. Clearly, however, it is a reflexive relation, as we must say that the information content in any step of the sequence is contained in that very step itself.

The third description brings in yet another distinct item of information, namely that the event described in step two is one that took place slowly. Thus the information content of step three includes that of step two. We have already established that two includes one. Now we can see that informational content inclusion in such a sequence is transitive, and we may conclude that the information in step three also includes that given in step one.

Similar reasoning applies to all seven steps in this sequence. Each subsequent step includes all of its predecessors. The seventh step is therefore related by informational inclusion to every step in the sequence. Thus the relation of informational inclusion effects a linear ordering of the sequence.

As a proposition that contains information about how an event happened, 'Smith buttered the toast slowly' is more specific, i.e., contains more information about how it was made true that the toast was buttered than 'Smith buttered the toast'. Slowly adds a new element, and tells you more about what happened and how it happened. In this sense of informational containment, we say that 'buttered slowly' contain 'buttered'.



We look at it this way insofar as we regard these two propositions as expressing different degrees of information about what was made to be true.

But there is an ambiguity. If we look at the different ways these two propositions could come to be true, it seems that the information that it is possible that Smith buttered the toast slowly is contained in the information that it is possible that Smith buttered the toast. It's not the case that every way in which he could butter the toast is a way in which he buttered it slowly, e.g., he could butter it quickly. But the converse is true: every way in which he butters the toast slowly is a way in which he butters the toast. In this sense 'Smith buttered the toast' contains more information about the possible ways things could have happened.

With a sequence like the one we began with however, it seems that our concern is more with what information is given about what was made true and how it was made true, than with the different possibilities of how it could have come to be true. However, we will return to this alternative notion of informational containment of possible ways things could have happened.

The general outline of our present approach is this. Given that we can start with such a set of bits of information about a sequence of actions, we can define the information content of any particular proposition made true at some stage as a subset of this large set we began with.[7] Let us call the set of bits of information about how each proposition was made true I. Then we are saying that $i(p)$ for any proposition made true in the sequence is some subset of I. Proper containment is not always required. Thus $i(p)$ can be I itself in some cases. Generally, if a sequence is made up of propositions $P_0, P_1, \ldots,$ $P_n,$ we will say that the informational content of that sequence is the union of the information content of all its components

$$\bigcup_{j=0}^{n} i_j.$$

The requirements we have now set down are sufficient to assure us that the correct approach to reasoning about internal action sequences can be modelled by a dependence logic of Epstein (1982). Following Epstein, we can introduce a binary relation of information-inclusion on the complex of propositions, called a *dependence relation,* where there is some I and i as above such that B is included in A if and only if $i(B) \subseteq i(A)$. Epstein's definition of a model takes into account both a truth valuation and a dependence relation. Consequently the way we want to define conditionals in an act-sequence as above is possible in a dependence logic. Some examples of tautologies are given below.

The kind of conditional involved, 'if A then B', requires both $\neg(A \wedge \neg B)$ and $i(B) \subseteq i(A)$ to be made true. The first requirement is satisfied by the pair of propositions; A = Bob swallowed a sword, B = 2 + 2 = 4. Presuming B is true then $\neg(A \wedge \neg B)$ is satisfied simply because B is true, $\neg\neg B$ is satisfied, and therefore $\neg(A \wedge \neg B)$ is also satisfied. The second requirement is satisfied by the pair of propositions;

A = Bob swallowed an imaginary sword, B = Bob swallowed a sword. All the information in B is contained in the information of A. Hence neither requirement alone is sufficient to determine the truth of 'If A then B' in the sense required for internal act-sequence inferences.

To determine that 'Bob swallowed a steel sword' forms the antecedent of a true conditional where 'Bob swallowed a sword' is the consequent, we must require both the right truth-values and the right information-inclusion relationship of these propositions. Neither by itself is adequate.

Neither informational containment nor truth-values of individual propositions is by itself sufficient to account for the kind of conditional involved in an internal act-sequence. Just using individual truth-values, we can truly say that if 'The toast is buttered' is allowed to be false then the following conditional must be true: whatever makes it true that the toast is buttered makes it true that Hannibal was defeated at the Trasimene Lake by the Romans. Just using information containment, we have to say that the information concerning how 'The toast is buttered' is made true is contained in the information on how 'The toast is buttered at midnight with a knife'. Yet clearly it is incorrect to venture that whatever makes the second proposition true makes the first proposition true. Thus the kind of conditional involved in internal act-sequences requires both $\neg(A \wedge \neg B)$ and $i(B) \subseteq i(A)$ to be made true.

Here are some examples of tautologies that result from this account of conditionals.

Some Tautologies of Dependence Logic

(1) $A \rightarrow [(A \rightarrow B) \rightarrow B]$	(10) $(A \rightarrow B) \rightarrow (\neg A \vee B)$
(2) $(A \wedge B) \rightarrow A$	(11) $A \rightarrow A$
(3) $(A \rightarrow B) \rightarrow [(B \rightarrow C) \rightarrow (A \rightarrow C)]$	(12) $(A \vee B) \rightarrow (B \vee A)$
(4) $(\neg A \vee \neg B) \rightarrow \neg(A \wedge B)$	(13) $A \rightarrow \neg\neg A$
(5) $(\neg A \wedge \neg B) \rightarrow \neg(A \vee B)$	(14) $\neg\neg A \rightarrow A$
(6) $(A \wedge \neg B) \rightarrow \neg(A \rightarrow B)$	(15) $\neg B \rightarrow [(A \rightarrow B) \rightarrow \neg A]$
(7) $\neg A \rightarrow [(A \vee B) \rightarrow B]$	(16) $(A \rightarrow B) \rightarrow [(A \wedge C) \rightarrow B]$
(8) $(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$	(17) $[A \rightarrow (B \wedge C)] \rightarrow [(A \rightarrow B) \wedge (A \rightarrow C)]$
(9) $(\neg B \rightarrow \neg A) \rightarrow (A \rightarrow B)$	(18) $A \rightarrow (\neg A \rightarrow A)$

To contrast the logic of internal versus external sequence of actions, note that (17) above holds in the dependence logic but fails in a relatedness logic. 'If the toast was buttered slowly at midnight, then the toast was buttered slowly and the toast was buttered at midnight' implies 'If the toast was buttered slowly at midnight then the toast was buttered slowly, and if the toast was buttered slowly at midnight then the toast was buttered at midnight.'

The general reasoning is as follows. Look at the consequent of (17). There are only four possible ways it could be allowed to be false: (i) if A is made true and B is allowed to be false, (ii) if B is not contained in A, (iii) if A is made true and C is allowed to be false, or (iv) if C is not contained in A. But each of these interpretations will make $A \rightarrow (B \wedge C)$, the antecedent of (17), allowed to be false. Clearly either (i) or (iii) would make $A \rightarrow (B \wedge C)$ false, just in virtue of the truth-values. But either of (ii) or (iv) would have the same effect, as we can see by considering (ii). If B is not contained in A, then clearly $B \wedge C$ will not be contained in A either. In short, every possible way of allowing the consequent of (17) to be false bars the possibility of making the antecedent true. Thus there is no consistent way to assign truth-values and information content assignments so that $A \rightarrow (B \wedge C)$ is made true and $(A \rightarrow B) \wedge (A \wedge C)$ is allowed to be false. So (17) is a tautology for content inclusion of act-sequences.

Yet for relatedness, (17) fails to be generally true. To disprove it, suppose A is related to B but not to C. Then $A \rightarrow C$ is allowed to be false, and hence $(A \rightarrow B) \wedge (A \rightarrow C)$ is allowed to be false. But $A \rightarrow (B \wedge C)$ could still be made true. Assume B and C both are allowed to be true. Then $A \rightarrow (B \wedge C)$ is allowed to be true, because A is related to $B \rightarrow C$ and B and C are allowed to be true. Then $A \rightarrow (B \wedge C)$ is made to be true but $(A \rightarrow B) \wedge (A \rightarrow C)$ is not. Thus (17) fails for relatedness of events. For example, let us assume that whatever makes it true that my finger is moved makes it true that the switch is flipped and the light is on. It need not follow that my moving my finger by itself makes it true that the light is on. It need not follow that just because I move my finger that by some sort of wizardry the light must go on.

Below are some formulas that are not tautologies in dependence logics.

Some Tautologies of Classical PC That Fail in Dependence Logics

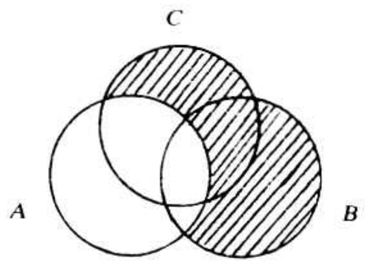
(1) $(A \wedge B) \rightarrow (A \rightarrow B)$	(7) $\neg(A \rightarrow B) \rightarrow (\neg A \wedge \neg B)$
(2) $A \rightarrow (A \vee B)$	(8) $\neg(A \wedge B) \rightarrow (\neg A \vee \neg B)$
(3) $A \rightarrow (\neg A \rightarrow B)$	(9) $[(A \wedge B) \rightarrow C] \rightarrow [(A \wedge \neg C) \rightarrow \neg B]$
(4) $A \rightarrow (B \rightarrow A)$	(10) $[(A \wedge B) \rightarrow C] \rightarrow [(A \rightarrow B) \vee (A \rightarrow C)]$
(5) $\neg(A \rightarrow B) \rightarrow (B \rightarrow A)$	(11) $A \rightarrow (A \rightarrow B)$
(6) $\neg(A \rightarrow B) \rightarrow (A \wedge \neg B)$	(12) $A \rightarrow (B \rightarrow A)$

Take (2) as an example. Whatever makes it true that the toast is buttered need not also make it true that the toast is buttered or the toast is eaten. Or consider (12). Whatever makes it true that the toast is buttered need not make it true that if the toast is eaten it is buttered.

As an illustration to show how Venn diagrams can be used along with truth-tables as a method of testing tautologies in dependence logic, let us look at the case of exportation. First consider

$$[A \rightarrow (B \rightarrow C)] \rightarrow [(A \wedge B) \rightarrow C]$$

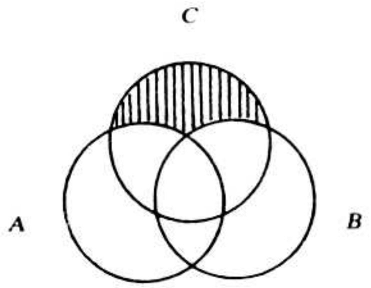
This schema is a classical tautology, but is it also a dependence tautology? For the antecedent to be true, it is required that $i(B \cup C) \subseteq$ $i(A)$, i.e.,



For the consequent to be true, it is required that $i(C) \subseteq i(A \cup B)$. This requirement has to be met, as the diagram shows that all information in C, but outside $A \cup B$, is nullified. Hence the above schema is a dependence tautology. What about the other way?

$$[(A \wedge B) \rightarrow C] \rightarrow [A \rightarrow (B \rightarrow C)]$$

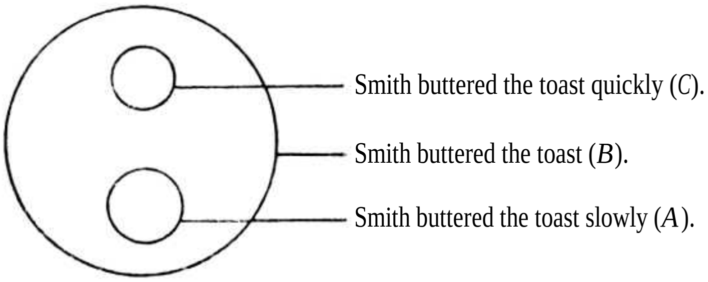
The antecedent requires $i(C) \subseteq i(A \cup B)$. But as the diagram below indicates, that (toes not mean that $i(B \cup C) \subseteq i(A)$.



There could be some information in $B \cup C$ that might not necessarily be included in the information in A. Thus by looking at all possible combinations of truth-values and all possible combinations of information overlap for the given set of atomic propositions, we can always determine whether a given wff is or is not a dependence tautology. The second schema fails to be a dependence tautology even though it is a tautology in classical propositional calculus.

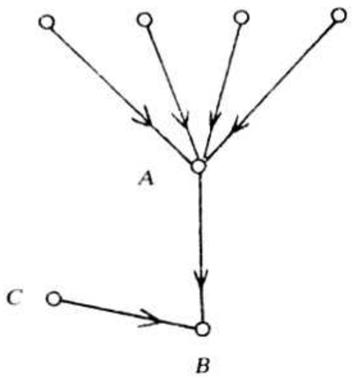
4. ALTERNATIVE LINES OF HISTORICAL VERIFICATION

Another thing we could mean by $A \subseteq B$ is that every way in which A could be known to have come to be true is also a way that B could be known to have come to be true. For example, every way in which 'Smith buttered the toast slowly' could have known to have come to be true is a way in which 'Smith buttered the toast' could have known to become to be true. But the converse is not true. For some ways Smith could have been known to have come to have buttered the toast, e.g., quickly. are not ways he could have been known to have come to butter it slowly. In this sense, 'Smith buttered the toast' contains more information about how things could have happened.



In this sense, the more general proposition contains more information about the different historically possible ways it could have become known to be true.

In this example there are four ways A could be known to have come to be true.



All the different ways that A could have become known to be true are also ways that B could have become known to be true.

According to this way of ordering the sequence of actions, the relation of information-inclusion is just the converse of the relation we have been studying in internal sequences. Hence the appropriate logic for this structure of internal sequences is a dual dependence logic of Epstein (1982). In keeping with this approach to actions, we might want to study internal sequences of actions by looking at the different world lines that represent the historical ways that the proposition could have been known to have come to be true. But note that this approach, the dual of our previous approach to internal act-sequences, tends to make the sequence more pluralistic. For if we are talking of Smith's buttering the toast slowly as opposed to quickly, both as being instances of buttering, it seems more plausible that we are talking of different historical possibilities of actions. Thus here the notion of an act-sequence yields information about different historical possibilities in a line of development. Really what we are investigating here is the epistemology of historical verification concerning how we come to know that a given action occurred. We leave the investigation of these conditionals for another occasion. It is enough to notice that this way of ordering actions is quite different from either of the previous two ways.

Some of the most fascinating open questions concern negation in act-sequences. In dual dependence logic, $i(\neg A) = i(A)$ and $i(A \wedge B) = i(A) \cup i(B)$. So (8), (9), and (16) in the list of tautologies for dependence logic will fail in dual dependence logic. Consider (8): $i(\neg A) =$

$i(A)$ and $i(\neg B) = i(B)$. Now we know in general that $i(A) \subseteq i(B)$ does not imply $i(B) \subseteq i(A)$ in dual dependence logic. Thus $i(A) \subseteq i(B)$ does not imply $i(\neg B) \subseteq i(\neg A)$ either. Hence $(A \to B) \to (\neg B \to \neg A)$ fails in dual dependency logic.

Parenthetically, we might note that this fact has important consequences for the Raven Paradox.[8] Every way it could become known to have come to be true that if Bob is a raven then Bob is black, need not be a way it could become to be known to be true that if Bob is not black then Bob is not a raven.

Epstein (1982) has worked out a number of logics in which the conditions $i(A \wedge B) = i(A) \cup i(B)$ and $i(\neg A) = \overline{i(A)}$ jointly obtain. The second condition means that, unlike all the systems we have considered above, $i(A) \neq i(\neg A)$. Moreover, in some modal logics, both conditions above are met. These developments open up new alternatives in studying the logic of act-sequences, and suggest (i) letting a proposition be false may, in some important sense, be less specific in information content than making the same proposition true, (ii) negation may be a promising avenue of new developments, and (iii) extensions to modal logic may bring out richer theories of the distinction between actions and omissions like the modal theory of Talja (1983). For the present, we now turn to a specific problem.

Cohen's problem of (1971, p. 63) can be stated as follows. Consider the following pair of conditionals.

(C1) If the government falls, there will be rioting in the streets.

(C2) If it is the case both that if the government falls there will be rioting in the streets, and also that the government will not fall, then the shopkeepers will be glad.

Let us say that (C1) and (C2) respectively have the forms '$A \to B$' and '$[(A \to B) \wedge \neg A] \to C$'. Now here is the problem. Assume A is false. Then by the truth-functional readings of $\neg$, $\wedge$, and $\to$, $A \to B$ and $\neg A$ are both true. Hence (C2) as a whole must be true. What is problematic, according to Cohen, is that the truth of the consequent of (C2), the statement that the shopkeepers will be glad, is assumed to be dependent on only one condition, the fate of the government.[9] Whereas really in asserting (C2), we are saying that it is dependent on two mutually independent conditions. What is wrong, according to Cohen (1971, p. 63), indicates that 'if ... then' cannot be truth-functional, even despite the attempts of conversationalist theorists to patch up the truth-functionalist analysis.

Cohen's problem does not arise on a relatedness analysis of (C1) and (C2), for letting A be false is not enough by itself to guarantee that $(A \rightarrow B) \wedge \neg A$ is allowed to be false. Such an assertion is only made if A is unrelated to B. Thus in the relatedness analysis, making C true is dependent on the two mutually independent conditions $A \rightarrow B$ and $\neg A$.

Moreover, other problems do not arise as well. Even if $A \rightarrow B$ and $\neg A$ were both allowed to be false, it still does not follow automatically that (C2) is true and that 'The shopkeepers will be glad' has to be made true regardless of its connections to $A \rightarrow B$ or $\neg A$. In order for (C2) to be made true, the consequent (C) does have to be related to at least one of the propositions in the antecedent.

By requiring relatedness of propositions made true or allowed to be false, relatedness logic provides an analysis of action inferences that is more comprehensive than an exclusively truth-functional analysis. In realistic argumentation of the *ad hominem* sort, the disputation often turns on relatedness or dependence relations that are quite complex. The links of relatedness in a sequence of actions is often itself a topic of dispute. Suppose Jones accuses Smith of selling weapons to a repressive regime. Then Smith replies, "Why, you're inconsistent and hypocritical. The university you are employed by has investments in companies that manufacture the very weapons used by this same regime." The allegation is that there is a sequence of links that could be filled in so that what Jones allows is a proposition related to some outcome equivalent to what Jones deplores. The allegation presumes an analysis that, if filled in, could support a demonstration of an act-theoretic inconsistency on the part of Jones. Whether the criticism of *ad hominem* hypocrisy is tenable or not must be fought out, we propose, by a filling in of the propositional logic once Smith and Jones agree on their premisses. However, we must leave the full analysis of such complex disputations for another study.

NOTES

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[1] See also Howard L. Dazeley and Wolfgang L. Gombocz: 1979, 'Interpreting Anselm as Logician'. *Synthese* **40**, 71-96.

[2] For further remarks on action propositions, see the author's critical notice of Pörn (1977) in *Synthese* **43**, 1980, 421-431.

[3] See the definition of a model for relatedness logic given by Epstein (1979, p. 148). The development in section 2 could be re-worked for a non-symmetrical $\mathscr{R}$, but we should note that contraposition will fail and $A \vee B$ will be defined as $\neg(\neg A \wedge \neg B) \wedge (R(A, B) \wedge R(B, A))$. Note that capital R is syntactic and script $\mathscr{R}$ is semantic.

[4] Some comparisons of relatedness logic and relevance logic are discussed in an article by the author, 'Philosophical Basis of Relatedness Logic', *Philosophical Studies* **36**, 1979, 115-136.

[4a] Symmetry of $\mathscr{R}$ could be dropped, except that it's needed to derive contraposition. The discussion in section 2 could easily be reworked for nonsymmetrical $\mathscr{R}$, but contraposition, (8), will fail.

[5] Failure of transitivity in $\mathscr{S}$ is closely connected to failure of a deduction theorem: in $\mathscr{S}$, just because we can prove B from A, it does not follow that $A \rightarrow B$ must obtain. In $\mathscr{S}$, if we can prove B from A and we can prove C from B, then we can always prove C from A. Yet transitivity for $\rightarrow$ does not obtain.

[6] However, Pörn also introduces other operators in his language that could possibly also be equated with our notion of making a proposition true. For further discussion see the reference of note 2 above.

[7] There will be many cases where it is difficult to straightforwardly assign an information set to a given proposition taken by itself, but given a set of propositions reconstructed from an act-sequence, we can argue relative information more easily.

[8] For a statement of the Raven Paradox, see Carl G. Hempel: 1965, 'Studies in the Logic of Confirmation', in C. G. Hempel, *Aspects of Scientific Explanation*, The Free Press, New York, pp. 3-51. Reprinted from *Mind* **54**, 1945, 1-26 and 97-121.

[9] You can see why by the following reasoning. If A is false then $(A \rightarrow B) \wedge \neg A$ must be true. If A is true then $(A \rightarrow B) \wedge \neg A$ must be false. Thus as Cohen (1971, p. 63) points out, evidence for the truth of (C2) is sufficient if it relates the fate of the government to the feelings of the shopkeepers "without having any bearing whatever on the causes and effects of rioting in the streets."

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